Assignment #3

Due on Wednesday, February 3, 2010

Read Section 1.1 on The Malthusian Model, pp. 2–5, in Allman and Rhodes.

Read Section 1.2 on Nonlinear Models, pp. 11–17, in Allman and Rhodes.

Do the following problems

1. Problem 1.1.11 on page 8 in Allman and Rhodes.

2. Problem 1.2.7 on page 18 in Allman and Rhodes.

3. Problem 1.2.8 on page 18 in Allman and Rhodes.

4. (US Census Data.) The MS Excel file CensusDataUS in the Math 36 webpage (see the courses website at http://pages.pomona.edu/~ajr04747) contains the total US population (in millions of people) for each year that a census has been taken in the United States.

(a) Use MATLAB® to get a plot of the US population as a function of $t$, where $t$ is in units of 10 years since the year 1790.

(b) If the US population follows a Malthusian model, what would the growth rate $\lambda$ be? Using this value of $\lambda$, compute the population values that the model predicts for $t = 1, 2, 3, \ldots$. Plot the predicted and actual values on the same graph. How well do these predictions compare with the actual data?

5. (US Census Data, continued). Starting with the solution to the Malthusian model: $N_t = N_0 \lambda^t$, take logarithms on both sides to get

$$\ln N_t = \ln N_0 + t \ln(\lambda).$$

Thus, the relationship between $\ln N_t$ and $t$ should be linear with slope $\ln(\lambda)$ and $y$–intercept $\ln N_0$.

(a) If $X$ represents a row of values, and $Y$ another row of values of the same size, the MATLAB® function polyfit(X,Y,1) returns the slope $m$ and $y$–intercept $y_o$ of the line that best fits the data (in the sense of least squares regression) in $X$ and $Y$: $y = mx + y_o$.

Use this function to obtain estimates for the values of $\ln N_0$ and $\ln(\lambda)$.

(b) Obtain estimates for $N_0$ and $\lambda$, and use them to generate a new set of predicted values for the US population. Plot these, along with the actual data, and assess how good the fit is.