Exam 2 (Part I)

Monday, April 5, 2010

Name: ____________________________________________

Show all significant work and justify all your answers. This is a closed book exam. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 2 problems. Relax.

1. Suppose that the rate at which a drug leaves the bloodstream and passes into the urine at a given time is proportional to the quantity of the drug in the blood at that time.

   (a) Write down and solve a differential equation for the quantity, \( Q = Q(t) \), of the drug in the blood at time, \( t \), in hours. State all the assumptions you make and define all the parameters that you introduce.

   (b) Suppose that an initial dose of \( Q_0 \) is injected directly into the blood, and that 20% of that initial amount is left in the blood after 3 hours. Based on the solution you found in the previous part, write down \( Q(t) \) for this situation and sketch its graph.

   (c) How much of the drug is left in the patient’s body after 6 hours if the patient is given 100 mg initially?

2. Suppose a bacterial colony has \( N_0 \) bacteria at time \( t = 0 \). Let \( M(t) \) denote the number of bacteria that develop certain mutation during the time interval \([0, t]\). Assume that, for small \( \Delta t > 0 \),

   \[
   M(t + \Delta t) - M(t) \approx a (\Delta t) N(t),
   \]

   where \( a \) is a positive constant, and \( N(t) \) is the number of bacteria in the colony at time \( t \).

   (a) Give an interpretation to what the expression in (1) is saying. In particular, provide a meaning for the constant, \( a \), known as the mutation rate.

   (b) Let \( \mu(t) = E(M(t)) \) denote the expected value of the number of mutations in the time interval \([0, t]\). It is possible to prove, using the expression in (1), that \( \mu = \mu(t) \) is differentiable and satisfies the differential equation

   \[
   \frac{d\mu}{dt} = aN(t).
   \]

   Solve the differential equation in (2) assuming that \( N(t) \) grows in time according to a Malthusian model with per–capita growth rate \( k \), and that there are no mutant bacteria at time \( t = 0 \).