Assignment #1

Due on Wednesday, January 26, 2011

Read Sections 2.1 on the *Definition of Euclidean Space* in the class Lecture Notes (pp. 7–8).
Read Sections 2.2 on *Spans, Lines and Places* in the class Lecture Notes (pp. 8–11).
Read Section 1.2 on *The Vector Space* $\mathbb{R}^n$ in Baxandall and Liebek's text (pp. 2–9).

Do the following problems

1. Let $\vec{v}_1 = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$.

   (a) Give the parametric equations of the line through the point $P\colon (0,4,7)$ in the direction of the vector $\vec{v}_1$.

   (b) Give the equation of the plane through the point $P\colon (0,4,7)$ spanned by the vectors $\vec{v}_1$ and $\vec{v}_2$.

2. The following give parametric equations to two lines in $\mathbb{R}^3$:

$$\left\{\begin{array}{rcl} x &=& -1+4t \\ y &=& -7t \\ z &=& 2-t \end{array}\right. \qquad \left\{\begin{array}{rcl} x &=& -1+s \\ y &=& 2-s \\ z &=& 2s \end{array}\right.$$

   Determine if the two lines ever meet. Justify your answer. If the lines do meet, give the equation of the plane that contains both lines.

3. The following give parametric equations to two lines in $\mathbb{R}^3$:

$$\left\{\begin{array}{rcl} x &=& 2+4t \\ y &=& -1-7t \\ z &=& 2-t \end{array}\right. \qquad \left\{\begin{array}{rcl} x &=& s \\ y &=& 1-s \\ z &=& -2+2s \end{array}\right.$$

   Determine if the two lines ever meet. Justify your answer. If the lines do meet, give the equation of the plane that contains both lines.

4. The vectors $\vec{v}_1 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\vec{v}_3 = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ in $\mathbb{R}^3$ can span a line, a plane or the entire three dimensional space $\mathbb{R}^3$. Give the equation of the geometric object which they span.

5. Consider the plane whose equation is

$$x - 4y + 7z = 3. \tag{1}$$

   (a) Find a vector perpendicular to this plane.

   (b) Find two vectors that span the plane given by the equation in (1).