Exam 2

April 29, 2011

Name: ______________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 5 problems. Relax.

1. Assume that the temperature in a region, U, of three–dimensional space is given by a function $f\colon \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x,y,z) = cx^2(y-z), \qquad \text{for all } (x,y,z) \in U,$$

   and some positive constant c.

   An insect flies in the region along a path modeled by a C^1 function $\sigma\colon \mathbb{R} \to \mathbb{R}^3$. Suppose that at time $t=1$ the insect is located at $(1,1,0)$ and its velocity is $\sigma'(1) = \widehat{i} - \widehat{j} + 2\widehat{k}$. Compute the rate of change of temperature sensed by the insect at time $t=1$. Is the temperature increasing or decreasing at that instant?

2. Set up the integral (but, **do not evaluate it**) that yields the arc–length of the ellipse, C, given by the graph of the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

   for positive real numbers a and b. Explain all the steps leading to your derivation of a formula for $\ell(C)$.

3. Let ω denote a differential 1–form in $\mathbb{R}^3$, and T the oriented triangle $[P_oP_1P_2]$ in $\mathbb{R}^3$

   (a) State the Fundamental Theorem of Calculus for the differential form ω acting on the the boundary, ∂T, of the oriented triangle T.

   (b) Apply the Fundamental Theorem of Calculus to evaluate the line integral

$$\int_{\partial T} y\ dx + 2x\ dy + z^2\ dz,$$

   where the vertices of T are $P_o(1,0,0)$, $P_1(0,1,0)$ and $P_2(0,0,1)$.

4. Let U denote an open subset in $\mathbb{R}^2$ and $F\colon U \to \mathbb{R}^2$ be C^1 vector field. Let C denote a simple closed curve in U.

   (a) Write $F(x,y) = P(x,y)\,\widehat{\imath} + Q(x,y)\,\widehat{\jmath}$, where P and Q denote C^1 scalar fields defined in U. Define the flux of F across the simple, closed curve C and give a formula for computing it as line integral over C.

   (b) Let R denote the parallelogram spanned by the vectors $\overrightarrow{OP_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\overrightarrow{OP_2} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, and let C denote the boundary, ∂R, of R oriented in the counterclockwise sense. Use the Fundamental Theorem of Calculus to evaluate the flux of the the field

   $$F(x,y) = 2x\,\widehat{\imath} + y\,\widehat{\jmath}$$

   across C.

5. Evaluate the double integral $\displaystyle\iint_R xy\,dxdy$, where R is the region in the xy–plane sketched in Figure 1.

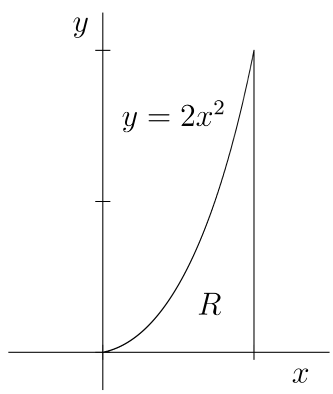

Figure 1: Sketch of Region R in Problem 5