Review Problems for Exam 2

(1) Let X and Y be independent Normal$(0, 1)$ random variables. Put $Z = \dfrac{Y}{X}$. Compute the distribution functions $F_Z(z)$ and $f_Z(z)$.

(2) A random point (X, Y) is distributed uniformly on the square with vertices $(-1, -1)$, $(1, -1)$, $(1, 1)$ and $(-1, 1)$.

   (a) Give the joint pdf for X and Y.

   (b) Compute the following probabilities: (i) $P(X^2 + Y^2 < 1)$, (ii) $P(2X - Y > 0)$, (iii) $P(|X + Y| < 2)$.

(3) Prove that if the joint cdf of X and Y satisfies

$$F_{X,Y}(x, y) = F_X(x) F_Y(y),$$

   then for any pair of intervals (a, b) and (c, d),

$$P(a < X \leq b, c < Y \leq d) = P(a < X \leq b) P(c < Y \leq d).$$

(4) The random pair (X, Y) has the joint distribution

X \ Y	2	3	4
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	$\frac{1}{6}$	0	$\frac{1}{3}$
3	$\frac{1}{12}$	$\frac{1}{6}$	0

   (a) Show that X and Y are not independent.

   (b) Give a probability table for random variables U and V that have the same marginal distributions as X and Y, respectively, but are independent.

(5) Let X denote the number of trials needed to obtain the first head, and let Y be the number of trials needed to get two heads in repeated tosses of a fair coin. Are A and Y independent random variables?

(6) Let $X \sim$ Normal$(0, 1)$ and put $Y = X^2$. Find the pdf for Y.

(7) Let X and Y be independent Normal$(0, 1)$ random variables. Compute $P(X^2 + Y^2 < 1)$.

(8) Suppose that X and Y are independent random variables such that $X \sim$ Uniform$(0, 1)$ and $Y \sim$ Exponential(1).

   (a) Let $Z = X + Y$. Find F_Z and f_Z.

   (b) Let $U = Y/X$. Find F_U and f_U.

(9) Let $X \sim$ Exponential(1), and define Y to be the integer part of $X + 1$; that is, $Y = i + 1$ if and only if $i \leq X < i + 1$, for $i = 0, 1, 2, \ldots$ Find the pmf of Y, and deduce that $Y \sim$ Geometric(p) for some $0 < p < 1$. What is the value of p?

(10) The expected number of typographical errors on a page of a certain magazine is 0.20. What is the probability that an article of 10 pages contains (a) no typographical errors, and (b) two or more typographical errors. Explain your reasoning.