Review Problems for Exam 3

(1) Suppose that a book with n pages contains on average λ misprints per page. What is the probability that there will be at least m pages which contain more than k missprints?

(2) Suppose that the total number of items produced by a certain machine has a Poisson distribution with mean λ, all items are produced independently of one another, and the probability that any given item produced by the machine will be defective is p.
Let X denote the number of defective items produced by the machine.

(a) Determine the marginal distribution of the number of X.

(b) Let Y denote the number of non–defective items produced by the machine. Show that X and Y are independent random variables.

(3) Suppose that the proportion of color blind people in a certain population is 0.005. Estimate the probability that there will be more than one color blind person in a random sample of 600 people from that population.

(4) An airline sells 200 tickets for a certain flight on an airplane that has 198 seats because, on average, 1% of purchasers of airline tickets do not appear for departure of their flight. Estimate the probability that everyone who appears for the departure of this flight will have a seat.

(5) Let X denote a positive random variable such that $\ln(X)$ has a Normal$(0,1)$ distribution.

(a) Give the pdf of X and compute its expectation.

(b) Estimate $\Pr(X \leq 6.5)$.

(6) Forty seven digits are chosen at random and with replacement from $\{0, 1, 2, \ldots, 9\}$. Estimate the probability that their average lies between 4 and 6.

(7) Let $X_1, X_2, \ldots, X_{30}$ be independent random variables each having a discrete distribution with pmf:

$$p(x) = \begin{cases} 1/4 & \text{if } x = 0 \text{ or } x = 2, \\ 1/2 & \text{if } x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Estimate the probability that $X_1 + X_2 + \cdots + X_{30}$ is at most 33.

(8) Roll a balanced die 36 times. Let Y denote the sum of the outcomes in each of the 36 rolls. Estimate the probability that $108 \leq Y \leq 144$.
Suggestion: Since the event of interest is $(Y \in \{108, 109, \ldots, 144\})$, rewrite $\Pr(108 \leq Y \leq 144)$ as

$$\Pr(107.5 < Y \leq 144.5).$$

(9) Let $Y \sim$ Binomial$(100, 1/2)$. Use the Central Limit Theorem to estimate the value of $\Pr(Y = 50)$.

(10) Let $Y \sim$ Binomial$(n, 0.55)$. Find the smallest value of n such that, approximately,

$$\Pr(Y/n > 1/2) \geq 0.95.$$

(11) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Thus, $Y = \displaystyle\sum_{i=1}^{n} X_i$ has a Poisson distribution with mean $n\lambda$. Moreover, by the Central Limit Theorem, $\overline{X} = Y/n$ has, approximately, a Normal$(\lambda, \lambda/n)$ distribution for large n. Show that $u(Y/n) = \sqrt{Y/n}$ is a function of Y/n which is essentially free of λ.