Assignment #6

Due on Monday, February 13, 2012

Read Section 3.2 on *Analysis of the Traffic Flow Equation* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`.

Do the following problems.

1. Find a solution to the initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = 0, & x \in \mathbf{R}, t > 0; \\ u(x,0) = f(x), & x \in \mathbf{R}, \end{cases}$$

where $f(x) = 1 - x^2$ for $0 \leqslant x \leqslant 1$, $f(x) = 0$ for $x > 1$ or $x < 0$. For various values of t, sketch the solution u as a function of x.

2. Find an implicit solution to the initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} - xu\dfrac{\partial u}{\partial x} = 0, & x \in \mathbf{R}, t > 0; \\ u(x,0) = x, & x \in \mathbf{R}. \end{cases}$$

3. In this problem we consider the equation $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$, where c is a real constant not equal to 0, in the region of the xt–plane determined by $x > 0$ and $t > 0$, and subject to the boundary condition

$$\begin{cases} u(x,0) = f(x) & x > 0 \\ u(0,t) = g(t) & t > 0, \end{cases}$$

where f and g are given continuous functions of a single variable.

   (a) Show that the boundary curve is not a characteristic of the equation.

   (b) If $c > 0$, determine a solution of the problem.

   (c) Show that if $c < 0$, then the problem in general cannot be solved.

4. Solve the initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} + x\dfrac{\partial u}{\partial x} = 1, & x \in \mathbf{R}, t > 0; \\ u(x,0) = e^x, & x \in \mathbf{R}. \end{cases}$$

5. Find the general solution to the linear partial differential equation

$$t\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = nu$$

where n is a positive integer.