Assignment #13

Due on Friday, March 15, 2013

Read Section 3.1, on *Vector Space Structure in* $\mathbb{M}(m,n)$, in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Background and Definitions

- *(Transpose of a matrix).* Given an $m \times n$ matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix},$$

  the **transpose** of A, denoted by A^T, is the $n \times m$ matrix given by

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}.$$

  More concisely, if $A = [a_{ij}]$, for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$, then

$$A^T = [a_{ji}], \quad \text{for } 1 \leqslant i \leqslant m \text{ and } 1 \leqslant j \leqslant n.$$

- *(Symmetric matrices).* A square matrix, $A \in \mathbb{M}(n,n)$, is said to be **symmetric** if $A^T = A$.

- *(Diagonal matrices).* A square matrix, $A = [a_{ij}] \in \mathbb{M}(n,n)$, is said to be a **diagonal** matrix if $a_{ij} = 0$ for all $i \neq j$.

Do the following problems

1. Let $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{M}(2,2) \;\middle|\; d = a \text{ and } c = -b \right\}$. Prove that W is a subspace of $\mathbb{M}(2,2)$.

2. Let W be as in Problem 1. Find a basis for W and compute $\dim(W)$.

3. Let $W = \{A \in \mathbb{M}(2,2) \mid A^T = A\}$; that is, W is the set of all 2×2 symmetric matrices. Prove that W is a subspace of $\mathbb{M}(2,2)$. Find a basis for W and compute its dimension.

4. Determine whether or not the set

$$\left\{ \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} -2 & 0 \\ 3 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -3 \\ 6 & -3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix} \right\}$$

forms a basis for $\mathbb{M}(2,2)$.

5. Let $W = \{A \in \mathbb{M}(n,n) \mid A \text{ is a diagonal matrix}\}$; that is,

$$A = [a_{ij}] \in W \text{ iff } a_{ij} = 0 \text{ for all } \ i \neq j.$$

Prove that W is a subspace of $\mathbb{M}(n,n)$ and compute $\dim(W)$.