Assignment #15

Due on Wednesday, April 3, 2013

Read Section 3.2, on *Matrix Algebra,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let A be an $m\times n$ matrix, and $\{e_1, e_2, \ldots, e_n\}$ denote the standard basis in $\mathbb{R}^n$.

   (a) Prove that Ae_j is the j^{th} column of the matrix A.

   (b) Use your result from part (a) to prove that $AI = A$, where I denotes the $n\times n$ identity matrix.

2. Recall that the null space of a matrix $A\in\mathbb{M}(m,n)$, denoted by N_A, is the space of solutions to the equation $Ax=\mathbf{0}$; that is, $N_A=\{v\in\mathbb{R}^n \mid Av=\mathbf{0}\}$. Prove that $v\in N_A$ if and only if v is orthogonal to the rows of A.

3. Recall that the transpose of an $m\times n$ matrix, $A=[a_{ij}]$, is the $n\times m$ matrix A^T given by $A^T=[a_{ji}]$, for $1\leqslant i\leqslant m$ and $1\leqslant j\leqslant n$.

   Let $A\in\mathbb{M}(m,n)$ and $B\in\mathbb{M}(n,k)$. Prove that $(AB)^T=B^TA^T$.

4. Consider any diagonal matrix $A=\begin{pmatrix} d_1 & 0 & 0\\ 0 & d_2 & 0\\ 0 & 0 & d_3\end{pmatrix}\in\mathbb{M}(3,3)$.

   Prove that there exist constants c_o , c_1 , c_2 and c_3 such that
   $$c_oI+c_1A+c_2A^2+c_2A^3=O,$$
   where I is the identity matrix in $\mathbb{M}(3,3)$ and O denotes the 3×3 zero–matrix. In other words, there exists a polynomial, $p(x)=c_o+c_1x+c_2x^2+c_3x^3$, of degree 3, such that $p(A)=O$.

5. Let $A=\begin{pmatrix} 1 & 2 & 1\\ 0 & -2 & 3\\ 4 & 1 & 2\end{pmatrix}$.

   (a) Compute A^2 and A^3.

   (b) Verify that $A^3-A^2-11A-25I=O$, where I is the identity matrix in $\mathbb{M}(3,3)$ and O denotes the 3×3 zero–matrix.

   (c) Use the result of part (b) above to find a matrix $B\in\mathbb{M}(3,3)$ such that $AB=I$.