Assignment #17

Due on Monday, April 8, 2013

Read Section 3.3, on *Invertibility,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Explain why the matrix
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$
is invertible, and use elementary row operations to compute its inverse.

2. Prove that, if $ad - bc \neq 0$, then the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible and compute A^{-1}.

3. A matrix $A \in \mathbb{M}(m, n)$ is said to be **singular** if the equation $Ax = \mathbf{0}$ has non–trivial solutions in $\mathbb{R}^n$.

   (a) Show that if $m < n$, then A is singular.

   (b) Prove that A is singular if and only if the columns of A are linearly dependent in $\mathbb{R}^m$.

4. A matrix $A \in \mathbb{M}(m, n)$ is said to be **nonsingular** if the equation $Ax = \mathbf{0}$ has only the trivial solution.

   (a) Prove that, if $A \in \mathbb{M}(m, n)$ is nonsingular, then the columns of A are linearly independent.

   (b) Deduce that, if $A \in \mathbb{M}(m, n)$ is nonsingular, then $\dim(C_A) = n$, where C_A denotes the column space of A; that is, C_A is the span of the columns of A.

5. Let A denote an $n \times n$ matrix. Prove that A is nonsingular if and only if A is invertible.