Assignment #23

Due on Friday, April 26, 2013

Read Section 4.6.2, on *Determinant of* 2×2 *matrices,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.6.4, on *The Cross-Product,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.6.5, on *The Triple Scalar Product,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.6.6, on *Determinant of* 3×3 *matrices,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Background and Definitions

Determinant of a 3×3 matrix. The determinant of the 3×3 matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

is defined to be

$$\det(A) = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}.$$

Geometrically, the absolute value of the determinant of A gives the volume of the parallelepiped determined by the columns of A.

Properties of the determinant of 3×3 matrices.

See Proposition 4.6.10 and Proposition 4.6.16 in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Compute $\det(A)$.

Based on your answer, what can you say about the matrix A?

2. Let
$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}.$$
Compute $\det(A)$.

Based on your answer, what can you say about the matrix A?

3. Given a vector $n = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3$, define the 3×3 matrix
$$A_n = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}.$$
   (a) Compute $\det(A_n)$ and deduce that A_n is singular.

   (b) Assume that $v \neq \mathbf{0}$ and compute the null space, $\mathcal{N}_{A_n}$, of A_n. Give a basis for $\mathcal{N}_{A_n}$ and compute $\dim(\mathcal{N}_{A_n})$.

4. Given a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the trace of A, denoted $\text{tr}(A)$, is defined to be $\text{tr}(A) = a + d$.

For any value λ, verify that
$$\det(A - \lambda I) = \lambda^2 - \text{tr}(A)\ \lambda + \det(A),$$
where I denotes the 2×2 identity matrix.

5. Let $A = \begin{pmatrix} 1 & -2 \\ 2 & 5 \end{pmatrix}$.

   (a) Use the result of Problem 4 to find a value of λ for which the equation
$$Av = \lambda v, \tag{1}$$
   has a nontrivial solution $v \in \mathbb{R}^2$.

   (b) For a value of λ found in part (a), give the solution space of the equation in (1) and compute its dimension.