Exam 1 (Part I)

Friday, March 8, 2013

Name: ____________________________________

This is a closed book exam. Show all significant work and provide reasoning for all your assertions. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. Answer the following questions as thoroughly as possible.

   (a) State precisely what it means for the subset, S, of $\mathbb{R}^n$ to be linearly independent.

   (b) Let W denote a subspace of $\mathbb{R}^n$ and B a subset of W. State precisely what it means for B to be a basis for W.

   (c) Define the dimension of a subspace, W, of $\mathbb{R}^n$.

   (d) Let W denote a subspace of $\mathbb{R}^n$ with ordered basis $B = \{w_1, w_2, \ldots, w_k\}$. For any vector, w in W, define $[w]_B$, the coordinates of w relative to B.

   (e) Given vectors v and w in $\mathbb{R}^n$, state what it means for v and w to be orthogonal.

2. Let S denote a subset of $\mathbb{R}^n$.

   (a) Give a definition of span(S).

   (b) Let v_1, v_2 and v_3 denote vectors in $\mathbb{R}^n$. Assume that $v_3 \in \text{span}(\{v_1, v_2\})$. Prove that
   $$\text{span}(\{v_1, v_2\}) = \text{span}(\{v_1, v_2, v_3\}).$$

3. Let W denote a subset of $\mathbb{R}^n$.

   (a) State precisely what it means for W to be a subspace of $\mathbb{R}^n$.

   (b) Let $\langle v, w\rangle$ denote the Euclidean inner product in $\mathbb{R}^n$. For a fixed vector u in $\mathbb{R}^n$, define the set
   $$W = \{w \in \mathbb{R}^n \mid \langle u, w\rangle = 0\}.$$
   Prove that W is a subspace of $\mathbb{R}^n$.