Exam 1 (Part II)

Friday, March 8, 2013

Name: ______________________________

This is the out–of–class portion of Exam 1. It is a closed–book and closed–notes exam. Students are expected to work individually on these problems. You may not consult with anyone, or the online notes in the course's website, or any other website. Show all significant work and provide reasoning for all your assertions.
Write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Monday, March 11, 2013.**

I have read and agree to these instructions. Signature: ____________________

1. Let W denote the solution space of the homogenous system

$$\begin{cases} x_1 - x_2 + 2x_4 = 0 \\ -x_1 + x_3 - 3x_4 = 0 \\ x_1 - 2x_2 + x_3 + x_4 = 0 \\ 2x_1 - x_2 - x_3 + 5x_4 = 0. \end{cases}$$

   (a) Find a basis for W and compute $\dim(W)$.

   (b) Define $V = \{v \in \mathbb{R}^4 \mid \langle v, w\rangle = 0, \text{ for all } w \in W\}$, where $\langle v, w\rangle$ denotes the Euclidean inner product of v and w in $\mathbb{R}^4$.
   Show that V is subspace of $\mathbb{R}^4$, give a basis for V, and compute $\dim(V)$.

2. Let $A = \begin{pmatrix} 1 & -1 & 1 & 2 \\ -1 & 0 & -2 & -1 \\ 0 & 1 & 1 & -1 \\ 2 & -3 & 1 & 5 \end{pmatrix}$, and denote by C_A the span of the columns of the matrix A.

   (a) Give a basis for C_A and compute $\dim(C_A)$.

   (b) Determine whether or not the vector $v = \begin{pmatrix} 4 \\ 7 \\ 7 \\ 4 \end{pmatrix}$ is in C_A.

   (c) Given an arbitrary vector $v = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$ in $\mathbb{R}^4$, determine conditions on x, y, z and w that will guarantee that $v \in C_A$.