Exam 2 (Part I)

Friday, May 3, 2013

Name: ______________________________

This is a closed book exam. Show all significant work and provide reasoning for all your assertions. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. Complete the following definitions:
   (a) The function $f\colon \mathbb{R}^n \to \mathbb{R}^m$ is linear if ...
   (b) An $n \times n$ matrix, A, is invertible if ...
   (c) An $m \times n$ matrix, A, is singular if ...
   (d) A scalar, λ, is an eigenvalue of the linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^n$ if ...
   (e) If λ is an eigenvalue of a linear transformation, $T\colon \mathbb{R}^n \to \mathbb{R}^n$, then the eigenspace of T corresponding to λ, $E_T(\lambda)$, is ...

2. Let Q denote an $n \times n$ matrix.
   (a) State what it means for Q to be an orthogonal matrix.
   (b) Show that if Q is orthogonal, then $|\det(Q)| = 1$.
   (c) Show that if Q is orthogonal, then Q is invertible and give a formula for computing Q^{-1}.

3. Define a linear transformation, $T\colon \mathbb{R}^2 \to \mathbb{R}^2$, which maps the standard basis vectors, e_1 and e_2, in $\mathbb{R}^2$ to the vectors

$$w_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \text{and} \quad w_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix},$$

   respectively.
   (a) Give the matrix representation, M_T, for T relative to the standard basis in $\mathbb{R}^2$.
   (b) Compute $\det(T)$. Does T preserve orientation?
   (c) Show that T is invertible and compute the inverse of T.
   (d) Verify that $\lambda = 1$ is an eigenvalue of T and compute the corresponding eigenspace.