Topics for Final Exam

1. **Vector Space Structure in Euclidean Space**

   1.1 Definition of n–Dimensional Euclidean Space

   1.2 Vector addition and scalar multiplication

   1.3 Spans

   1.4 Linear independence

   1.5 Subspaces of Euclidean Space

   1.6 Bases

   1.7 Dimension

   1.8 Coordinates

2. **Connections with the Theory of Linear Equations**

   2.1 Homogeneous systems

   2.2 Fundamental theorem for homogenous systems of linear equations

   2.3 Nonhomogeneous systems

3. **Euclidean Inner Product and Norm**

   3.1 Row–column product and the Euclidean inner product

   3.2 Euclidean norm

   3.3 Orthogonality

   3.4 Orthonormal bases

4. **Matrices**

   4.1 The set, $\mathbb{M}(m,n)$, of $m \times n$ matrices as a linear space

   4.2 Matrix Algebra

   4.3 Null space, column space and row space of a matrix

   4.4 Elementary matrices and invertibility

   4.5 Singular and nonsingular matrices

5. **Linear Transformations**
   - 5.1 Definition of linearity
   - 5.2 Matrix representation
   - 5.3 Null space and image
   - 5.4 Compositions
   - 5.5 Invertible linear transformation
   - 5.6 Orthogonal transformations
     - 5.6.1 Orthogonal matrices
     - 5.6.2 Determinant, cross–product and triple–scalar product
     - 5.6.3 Area and volume preserving transformations
     - 5.6.4 Orientation preserving transformations
     - 5.6.5 Orthogonal, orientation preserving transformations in $\mathbf{R}^2$ and $\mathbf{R}^3$.

6. **The Eigenvalue Problem**
   - 6.1 Eigenvalues, eigenvectors and eigenspaces
   - 6.2 The eigenvalue problem
   - 6.3 Application: Orthogonal transformations in $\mathbf{R}^2$ and $\mathbf{R}^3$.

Relevant sections in text: 1.5, 1.6, 1.8, 1.9, 1.10, 2.1 and 2.2.

Relevant chapters in the online class notes: Chapters 2, 3, 4 and 5

Important Concepts: Euclidean space; linear independence; span; subspaces; bases; dimension; coordinates; inner product; norm; orthogonality; linear transformation; null space; image; elementary matrices; invertible matrices and linear transformations; eigenvalue, eigenvector and eigenspace of linear transformations.

Important Skills: Know how to determine whether subsets of $\mathbf{R}^n$ are linearly independent; know how to tell whether a given subset of $\mathbf{R}^n$ is a subspace; know how to tell whether a set of vectors in $\mathbf{R}^n$ spans a subspace; know how to compute the span of a set of vectors; know how to solve systems of linear equations; know how to determine bases for subspaces of Euclidean space; know how to compute dimensions of subspaces; know how to find coordinates of vectors relative to ordered bases; know how to tell whether vectors are orthogonal; know how to tell whether a given matrix is invertible or not; know how to compute inverses of invertible matrices; know how to determine whether a given function is linear or not; know how to obtain matrix representations of linear transformations; know how to compute determinants of 2×2 and 3×3 matrices; know how to find eigenvalues, eigenvectors and eigenspaces of linear transformations.