

Assignment #11

Due on Monday, March 3, 2014

Read Section 4.1 on *Expected Value of a Random Variable* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 4.1 on *The Expectation of a Random Variable* in DeGroot and Schervish.

Do the following problems

1. Two discrete random variable, X and Y , are said to be **independent** if

$$\Pr(X = x, Y = y) = \Pr(X = x) \cdot \Pr(Y = y)$$

for all possible values of x and y or X and Y , respectively.

Prove that if X and Y are discrete and independent, then

$$E(X + Y) = E(X) + E(Y).$$

2. Let X be a discrete random variable with pmf $p_X(x)$, and assume that $p_X(x)$ is positive at $x = -1, 0, 1$ and zero elsewhere.

(a) If $p_X(0) = \frac{1}{4}$, find $E(X^2)$.

(b) If $p_X(0) = \frac{1}{4}$ and if $E(X) = \frac{1}{4}$, determine $p_X(-1)$ and $p_X(1)$.

3. A bowl contains 10 chips, of which eight are marked \$2 and two are marked \$5 each. Let a person choose, at random and without replacement, three chips from the bowl. If the person is to receive the sum of the resulting amounts, find this expectation.

4. Let $p_X(k) = \left(\frac{1}{2}\right)^k$, for $k = 1, 2, 3, \dots$, zero elsewhere, be the pmf of a discrete random variable X . Find the mean value of X .

Hint: For $|t| < 1$, define the function $f(t) = \sum_{k=0}^{\infty} t^k$. This is a geometric series

which adds up to $\frac{1}{1-t}$. Compute $f'(t)$.

5. An experiment consists of tossing a balanced die until a 6 comes up. On average, how many tosses are required to get a 6? In other words, if X denotes the number of tosses it takes to get a 6, what is $E(X)$? Show your calculations and justify your reasoning.