

Assignment #16

Due on Friday, March 14, 2014

Read Section 5.1 on the *Definition of the Joint Distribution* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 5.2 on *Marginal Distributions* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 3.4 on *Bivariate Distributions* in DeGroot and Schervish.

Read Section 3.5 on *Marginal Distributions* in DeGroot and Schervish.

Do the following problems

- Suppose that in an electric display sign there are three light bulbs in the first row and four light bulbs in the second row. Let X denote the number of bulbs in the first row that will be burned out at a specified time t , and let Y denote the number of bulbs in the second row that will be burned out at the same time t . Suppose that the joint pmf of X and Y is as specified in Table 1:

| $X \backslash Y$ | 0 | 1 | 2 | 3 | 4 |
|------------------|------|------|------|------|------|
| 0 | 0.08 | 0.07 | 0.06 | 0.01 | 0.01 |
| 1 | 0.06 | 0.10 | 0.12 | 0.05 | 0.02 |
| 2 | 0.05 | 0.06 | 0.09 | 0.04 | 0.03 |
| 3 | 0.02 | 0.03 | 0.03 | 0.03 | 0.04 |

Table 1: Joint Probability Distribution for X and Y , $p_{(x,y)}$

Determine each of the following probabilities:

- (a) $\Pr(X = 2)$ (b) $\Pr(Y \geq 2)$ (c) $\Pr(X \leq 2 \text{ and } Y \leq 2)$
 (d) $\Pr(X = Y)$ (e) $\Pr(X > Y)$

- Suppose that X and Y have a continuous joint distribution for which the pdf is defined as follows: $f(x, y) = \begin{cases} cy^2 & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$

Determine

- (a) the value of c ; (b) $\Pr(X + Y > 2)$; (c) $\Pr(Y < 1/2)$;
 (d) $\Pr(X \leq 1)$; (e) $\Pr(X = 3Y)$.

3. Suppose a point X is chosen at random from a region S in the xy -plane containing all points (x, y) such that $x \geq 0$, $y \geq 0$, and $4y + x \leq 4$.
- (a) Determine the joint pdf of X and Y .
 - (b) Suppose that S_o is a subset of the region S having area α , and determine $\Pr[(X, Y) \in S_o]$.
4. Suppose that X and Y have a discrete distribution for which the joint pmf is defined as follows:

$$p_{(X,Y)}(x, y) = \begin{cases} \frac{1}{30}(x + y) & \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal pmfs of X and Y .
 - (b) Are X and Y independent?
5. Suppose the joint pdf of X and Y is as follows:

$$f_{(X,Y)}(x, y) = \begin{cases} \frac{15}{4}x^2 & \text{for } 0 \leq y \leq 1 - x^2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal pdfs of X and Y .
- (b) Are X and Y independent?