

Assignment #17

Due on Friday, April 4, 2014

Read Section 5.3 on the *Independent Random Variables* and Section 6.1 on *The Normal Distribution* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 3.5 on *Marginal Distributions* in DeGroot and Schervish.

Read Section 5.6 on *The Normal Distribution* in DeGroot and Schervish.

Do the following problems

1. Suppose X and Y are independent and let $g_1(X)$ and $g_2(Y)$ be functions for which $E(g_1(X)g_2(Y))$ exists. Show that

$$E(g_1(X)g_2(Y)) = E(g_1(X)) \cdot E(g_2(Y))$$

Conclude therefore that if X and Y are independent and $E(|XY|)$ is finite, then

$$E(XY) = E(X) \cdot E(Y).$$

2. Suppose X and Y are independent random variables for which the moment generating functions exist on some common interval of values of t . Show that

$$\psi_{X+Y}(t) = \psi_X(t) \cdot \psi_Y(t)$$

for t is the given interval.

3. Suppose that $X \sim \text{Normal}(\mu, \sigma^2)$ and define $Y = \frac{X - \mu}{\sigma}$.

Prove that $Y \sim \text{Normal}(0, 1)$

4. Let X_1 and X_2 denote independent, $\text{Normal}(0, \sigma^2)$ random variables, where $\sigma > 0$. Define the random variables

$$\bar{X} = \frac{X_1 + X_2}{2} \quad \text{and} \quad Y = \frac{(X_1 - X_2)^2}{2\sigma^2}.$$

Determine the distributions of \bar{X} and Y .

Suggestion: To obtain the distribution for Y , first show that

$$\frac{X_1 - X_2}{\sqrt{2} \sigma} \sim \text{Normal}(0, 1).$$

5. Let X_1 , X_2 , \bar{X} and Y be as in the previous problem. Prove that \bar{X} and Y are independent.

Suggestion: Start working on this problem as soon as possible!