

Assignment #18

Due on Monday, April 7, 2014

Read Section 6.1 on *The Normal Distribution* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 5.6 on *The Normal Distribution* in DeGroot and Schervish.

Do the following problems

1. Let $X \sim \text{Normal}(0, 1)$ and put $Y = X^2$. Find the pdf for Y .
2. Let X and Y be independent $\text{Normal}(0, 1)$ random variables. Put $Z = X + Y$. Compute the pdf of Z .
3. Let X and Y be independent $\text{Normal}(0, 1)$ random variables. Compute $\Pr(X^2 + Y^2 < 1)$.
4. Let $X_1, X_2, X_3, \dots, X_n$ be independent identically distributed $\text{Normal}(0, 1)$ random variables. Define

$$Y = X_1 + X_2 + \dots + X_n.$$

Use moment generating functions to determine the distribution of Y .

5. Two instruments are used to measure the height, h , of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$.

Let X_1 denote the measurement made by the first instrument and X_2 the measurement made by the second instrument. Assume that X_1 and X_2 are independent random variables, and let $X = \frac{X_1 + X_2}{2}$, the average of the two instruments.

- (a) Determine the distribution of X .
- (b) Compute the probability that their average of the two measurements is within $0.005h$ of the height of the tower?