Assignment #21

Due on Monday, April 14, 2014

Read Section 7.2 on the *mgf Convergence Theorem* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 7.3 on the *Central Limit Theorem* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.3 on *The Central Limit Theorem* in DeGroot and Schervish.

Do the following problems

1. Let $X_1, X_2, X_3, \ldots$ denote a sequence of independent, identically distributed random variables with mean μ. Assume that the moment generating function of X_1 exists in some interval around 0. Use the mgf Convergence Theorem to show that the sample means, $\overline{X}_n$, converge in distribution to a limiting distribution with pmf $p(x) = \begin{cases} 1 & \text{if } x = \mu; \\ 0 & \text{elsewhere.} \end{cases}$

2. Let $Y_n \sim \text{Binomial}(n, p)$, for $n = 1, 2, 3, \ldots$, and define $Z_n = \dfrac{Y_n - np}{\sqrt{np(1-p)}}$ for $n = 1, 2, 3, \ldots$ Use the Central Limit Theorem to find the limiting distribution of Z_n.

   Suggestion: Recall that Y_n is the sum of n independent Bernoulli(p) trials.

3. Suppose that 75% of the people in a certain metropolitan area live in the city and 25% of the people live in the suburbs. If 1200 people attending certain concert represent a random sample from the metropolitan area, what is the probability that the number of people from the suburbs attending the concert will be fewer than 270? State any assumption that make in your solution to this problem.

4. Suppose that a random sample of size n is to be taken from a distribution for which the mean is μ and the standard deviation is 3. Use the Central Limit Theorem to determine approximately the smallest value of n for which the following relation will be satisfied: $\Pr(|\overline{X}_n - \mu| < 0.3) \geqslant 0.95$.

5. Let X_n be a random variable having a binomial distribution with parameters n and p_n. Assume that $\lim_{n\to\infty} np_n = \lambda$. Prove that the mgf of X_n converges to the mgf of a Poisson distribution with parameter λ as $n \to \infty$.