

Assignment #26

Due on Friday, April 25, 2014

Read Chapter 8 on *Introduction to Estimation* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 7.2 on *The Chi-Square Distribution* in DeGroot and Schervish.

Do the following problems

1. Let X and Y be independent $\chi^2(1)$ distributions. Put $W = X + Y$. Use moment generating functions to find the distribution of W . Give the pdf of W .
2. Let X and W be independent random variables with moment generating functions ψ_X and ψ_W , respectively, defined on a common interval around 0, $|t| < \delta$, for some $\delta > 0$. Put $Y = X + W$. Express the mgf of W in terms of the mgf of X and the mgf of Y , for $|t| < \delta$.
3. Assume that X and Y are independent random variables. Prove that X and Y^2 are also independent random variables.
4. Assume that X and Y are independent random variables. Let a and b denote real numbers with $a \neq 0$. Prove that X and $aY + b$ are also independent random variables.
5. A we say that x_M is a mode of continuous random variable, X , if

$$f_X(x_M) = \max_{-\infty < x < \infty} f_X(x). \quad (1)$$

That is, $f_X(x_M)$ gives the largest possible value of the pdf of X . If there is only one value of x_M for which (1) holds true, we call x_M the mode of the distribution.

Assume that $X \sim \chi^2(4)$. Compute the mode of the distribution of X .