

## Assignment #9

Due on Monday, February 17, 2014

Read Chapter 3 *Random Variables* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 3.2 on *Continuous Distributions* in DeGroot and Schervish.

Do the following problems

1. Suppose the pdf of a random variable  $X$  is as follows:

$$f(x) = \begin{cases} \frac{4}{3}(1 - x^3) & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Sketch the pdf and determine the values of the following probabilities:

- (a)  $\Pr\left(X < \frac{1}{2}\right)$   
(b)  $\Pr\left(\frac{1}{4} < X < \frac{3}{4}\right)$   
(c)  $\Pr\left(X > \frac{1}{3}\right)$

2. Suppose the pdf of a random variable is as follows:

$$f(x) = \begin{cases} cx^2 & \text{for } 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the value of  $c$  and sketch the pdf.  
(b) Find the value of  $\Pr(X > 3/2)$ .
3. Let  $\mathcal{C} = \{x \in \mathbb{R} \mid 0 < x < \infty\}$  and  $\mathcal{B}$  denote the Borel sets in  $\mathcal{C}$ . Let the pdf of a random variable,  $X$ , defined on  $\mathcal{C}$  be given by

$$f_X(x) = e^{-x} \quad \text{for all } x > 0.$$

Let  $E_k = \{x \in \mathcal{C} \mid 2 - 1/k < x \leq 3\}$  for  $k = 1, 2, 3, \dots$

Compute  $\Pr(E_n)$  for all  $n$ , and  $\lim_{n \rightarrow \infty} \Pr(E_n)$ .

4. A point is selected at random from the sample space  $\mathcal{C} = \{x \in \mathbb{R} \mid 0 < x < 10\}$ . For any Borel subset  $E \subseteq \mathcal{C}$  the probability of  $E$  is defined to be

$$\Pr(E) = \int_E \frac{1}{10} dx.$$

Define  $X: \mathcal{C} \rightarrow \mathbb{R}$  to be

$$X(x) = x^2 \quad \text{for all } x \in \mathcal{C}.$$

Find the cumulative distribution function and the probability density function of  $X$ .

5. A *median* of the distribution of a random variable  $X$  is a value  $m$  for  $x$  such that

$$\Pr(X < m) \leq \frac{1}{2} \quad \text{and} \quad \Pr(X \leq m) \geq \frac{1}{2}.$$

If there is only one such value  $m$ , it is called *the median of the distribution*.

Suppose the pdf of a random variable  $X$  is given by the function

$$f(x) = \begin{cases} \frac{1}{8}x & \text{for } 0 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

Compute a median for the distribution of  $X$ . Is it the median of the distribution?