Exam 1 (Part II)

This is the out–of–class portion of Exam 1. This is a closed–book and closed–notes exam; you may consult only the "Special Distributions" handout. You may work on these questions as long as you wish. Show all significant work and give reasons for all your answers. Please, write your name on this page and staple it to your solutions.

Name: ______________________________

Due on Monday, February 24, 2014

1. Let E_1, E_2 and E_3, denote three events in a probability space, $(\mathcal{C}, \mathcal{B}, \Pr)$. We say that E_1, E_2 and E_3 are **mutually independent** if and only if
$$\Pr(E_i \cap E_j) = \Pr(E_i) \cdot \Pr(E_j), \quad \text{for } i \neq j, \tag{1}$$
and
$$\Pr(E_1 \cap E_2 \cap E_3) = \Pr(E_1) \cdot \Pr(E_2) \cdot \Pr(E_3). \tag{2}$$
   (a) Flip a fair coin twice in a row. Let E_1 denote the event that the first toss yields a head, E_2 the event that the second toss yields a head, and E_3 the event that both tosses yield the same outcome. Verify that E_1, E_2 and E_3 are not mutually independent. Which of (1) or (2) fails in this case?
   (b) Toss a balanced die twice in a row. Let E_1 denote the event that the first toss yields either a 1, or a 2, or a 3; E_2 the event that the first toss yields a 3, or a 4, or a 5; and E_3 the event that the sum of the outcomes of the two tosses is 9. Verify that E_1, E_2 and E_3 are not mutually independent. Which of (1) or (2) fails in this case?

2. Let T denote service time for customers coming through a checkout counter in a retail store; this is the time it takes for service to be completed at the checkout counter. We have seen in class and in the lecture notes that T has an Exponential(β) distribution, where β is the average time needed to complete service, measured in minutes.
   (a) Compute the probability that it will take at least t minutes for the transaction to be completed.
   (b) Explain why $(T > t)$ denotes the event that at time t the costumer is still being served.
   (c) Compute the probability that a costumer is still being served three minutes after the start of service.
   (d) Compute the probability that, given that a costumer has been at the checkout counter for three minute, service will be completed t minutes later.