Exam 3 (Part II)

Due on Monday, May 5, 2014

Name: _____

This is the out–of–class portion of Exam 3. There are three questions in this portion of the exam. This is a closed–book and closed–notes exam; you may consult only the "Special Distributions" and the "Normal Distribution Probabilities Table" handouts. You may work on these questions as long as you wish. Show all significant work and give reasons for all your answers.

Students are expected to work individually on these problems. You may not consult with anyone.

Please, write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Monday, May 5, 2014.**

I have read and agree to these instructions. Signature: _____

1. Let $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$, where λ_1 and λ_2 are positive parameters, be independent random variables. Define $W = X + Y$.

   (a) Determine the distribution of W.

   (b) Determine the conditional distribution

   $$\Pr[X = k \mid W = n], \quad \text{for } k = 0, 1, 2, \ldots, n, \text{ and } n = 1, 2, 3, \ldots.$$

2. A random experiment consists of tossing a coin 1000 times and determining the number of heads that come up in the 1000 tosses. Assume that the probability of a head in each toss is p, for $0 < p < 1$. Let Y denote the number of heads observed in the experiment.

   (a) Give the distribution of Y.

   (b) Assuming the coin in the experiment is fair, estimate the probability that at most 470 of the tosses will yield heads.

   (c) Suppose that 1000 tosses of a coin (with unknown probability of heads) yields an outcome of 470 or fewer heads. Based on your answer to part (b), how confident are you that the coin used in the experiment is a fair coin? Explain the reasoning leading to your answer.

3. Suppose that the number of costumers arriving at a store per day follows a Poisson distribution with parameter 50. Estimate the probability that, on a given month, at least 1470 costumers will visit the store. Explain the reasoning leading to your answer.