

Review Problems for Exam 3

1. Suppose that a book with n pages contains on average λ misprints per page. What is the probability that there will be at least m pages which contain more than k missprints?
2. Suppose that the total number of items produced by a certain machine has a Poisson distribution with mean λ , all items are produced independently of one another, and the probability that any given item produced by the machine will be defective is p .

Let X denote the number of defective items produced by the machine.

- (a) Determine the marginal distribution of the random variable X .
 - (b) Let Y denote the number of non-defective items produced by the machine. Show that X and Y are independent random variables.
3. Suppose that the proportion of color blind people in a certain population is 0.005. Estimate the probability that there will be more than one color blind person in a random sample of 600 people from that population.
 4. An airline sells 200 tickets for a certain flight on an airplane that has 198 seats because, on average, 1% of purchasers of airline tickets do not appear for departure of their flight. Estimate the probability that everyone who appears for the departure of this flight will have a seat.
 5. Let X denote a positive random variable such that $\ln(X)$ has a Normal(0, 1) distribution.
 - (a) Give the pdf of X and compute its expectation.
 - (b) Estimate $\Pr(X \leq 6.5)$.
 6. Forty seven digits are chosen at random and with replacement from $\{0, 1, 2, \dots, 9\}$. Estimate the probability that their average lies between 4 and 6.
 7. Let X_1, X_2, \dots, X_{30} be independent random variables each having a discrete distribution with pmf:

$$p(x) = \begin{cases} 1/4, & \text{if } x = 0 \text{ or } x = 2; \\ 1/2 & \text{if } x = 1; \\ 0 & \text{otherwise.} \end{cases}$$

Estimate the probability that $X_1 + X_2 + \dots + X_{30}$ is at most 33.

8. Roll a balanced die 36 times. Let Y denote the sum of the outcomes in each of the 36 rolls. Estimate the probability that $108 \leq Y \leq 144$.

Suggestion: Since the event of interest is $(Y \in \{108, 109, \dots, 144\})$, rewrite $\Pr(108 \leq Y \leq 144)$ as

$$\Pr(107.5 < Y \leq 144.5).$$

9. Let $Y \sim \text{Binomial}(100, 1/2)$. Use the Central Limit Theorem to estimate the value of $\Pr(Y = 50)$.

Suggestion: Observe that

$$\Pr(Y = 50) = \Pr(49.5 < Y \leq 50.5),$$

since Y is discrete.

10. Let $Y \sim \text{Binomial}(n, 0.55)$. Find the smallest value of n such that, approximately,

$$\Pr(Y/n > 1/2) \geq 0.95.$$

11. Let X_1, X_2, \dots, X_n be a random sample from a Poisson distribution with mean λ . Thus, $Y = \sum_{i=1}^n X_i$ has a Poisson distribution with mean $n\lambda$. Moreover, by the Central Limit Theorem, $\bar{X} = Y/n$ has, approximately, a $\text{Normal}(\lambda, \lambda/n)$ distribution for large n . Show that $u(Y/n) = \sqrt{Y/n}$ is a function of Y/n which is essentially free of λ .

12. Suppose that factory produces a number X of items in a week, where X can be modeled by a random variable with mean 50. Suppose also that the variance for a week's production is known to be 25. What can be said about the probability that this week's production will be between 40 and 60?

13. Let (X_n) denote a sequence of nonnegative random variables with means $\mu_n = E(X_n)$, for each $n = 1, 2, 3, \dots$. Assume that $\lim_{n \rightarrow \infty} \mu_n = 0$. Show that X_n converges in probability to 0 as $n \rightarrow \infty$.

14. Let X_1, X_2, \dots, X_n be a random sample from a Poisson distribution with mean λ . Thus, $Y_n = \sum_{i=1}^n X_i$ has a Poisson distribution with mean $n\lambda$. Moreover, by the Central Limit Theorem, $\bar{X}_n = Y_n/n$ has, approximately, a $\text{Normal}(\lambda, \lambda/n)$ distribution for large n . Show that, for large values of n , the distribution of $2\sqrt{n} \left(\sqrt{\frac{Y_n}{n}} - \sqrt{\lambda} \right)$ is independent of λ .