Assignment #12

Due on Wednesday, March 5, 2014

Read Section 3.1 on *Using Characteristic Curves* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Appendix A, on *First–Order Equations*, in Chapter 3 of the text, pp. 277–308.

Do the following problems

1. Find a solution to the initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = 0, & x \in \mathbb{R}, t > 0; \\ \\ u(x,0) = f(x), & x \in \mathbb{R}, \end{cases}$$

   where $f(x) = 1 - x^2$ for $-1 \leqslant x \leqslant 1$, $f(x) = 0$ for $|x| > 1$. For various values of t, sketch the solution u as a function of x.

2. Find an implicit solution to the initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} - xu\dfrac{\partial u}{\partial x} = 0, & x \in \mathbb{R}, t > 0; \\ \\ u(x,0) = x, & x \in \mathbb{R}. \end{cases}$$

3. In this problem we consider the equation $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$, where c is a real constant not equal to 0, in the region of the xt–plane determined by $x > 0$ and $t > 0$, and subject to the boundary condition

$$\begin{cases} u(x,0) = f(x) & x > 0 \\ u(0,t) = g(t) & t > 0, \end{cases}$$

   where f and g are given continuous functions of a single variable.

   (a) Show that the boundary curve is not a characteristic of the equation.

   (b) If $c > 0$, determine a solution of the problem.

   (c) Show that if $c < 0$, then the problem in general cannot be solved.

4. Solve the initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} + x\dfrac{\partial u}{\partial x} = 1, & x \in \mathbb{R}, t > 0; \\[2ex] u(x,0) = e^x, & x \in \mathbb{R}. \end{cases}$$

5. Find the general solution to the linear partial differential equation

$$t\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = nu$$

where n is a positive integer.