

Assignment #6

Due on Monday, February 16, 2015

Read Chapter 3 on *Solving Linear Systems* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 2.2, on *The Geometry of Systems*, in Blanchard, Devaney and Hall.

Do the following problems

1. For the following linear system, give the equations for the solution curves and sketch the phase portrait.

$$\begin{cases} \dot{x} = -x + y; \\ \dot{y} = -x - y. \end{cases}$$

2. Consider the system

$$\begin{cases} \frac{dx}{dt} = y; \\ \frac{dy}{dt} = -\omega^2 x, \end{cases} \quad (1)$$

where ω is a positive constant.

- (a) Use the change of variables

$$u = \frac{1}{\omega}y$$

$$v = x$$

to turn system (1) into a system in the u and v variables.

- (b) Solve the system in part (a) and use it to construct solutions to the system in (1).
- (c) Sketch the phase-portrait of the system in (1).

3. Sketch the phase portrait of the system

$$\begin{cases} \frac{dx}{dt} = -x + 4y; \\ \frac{dy}{dt} = -2x + 3y. \end{cases}$$

4. Turn the second order equation

$$x'' + x = 0 \tag{2}$$

into a two-dimensional linear system; construct a solution to the system; and use the solution of the system to construct solutions to (3).

Give a solution to (3) subject to the initial conditions $x(0) = 0$, $x'(0) = 1$.

5. Turn the second order equation

$$x'' - x' - 2x = 0 \tag{3}$$

into a two-dimensional linear system; construct a solution to the system; and use the solution of the system to construct solutions to (3).

Give a solution to (3) subject to the initial conditions $x(0) = 1$, $x'(0) = 0$.