Assignment #11

Due on Friday, March 6, 2015

Read Section 14.3, on *Local Linearity and the Differential,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

Read Section 14.4, on *Gradients and Directional Derivatives in the Plane,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

Background and Definitions.
Directional Derivative. Let $f\colon D\to\mathbb{R}$ be a real–valued function defined on some domain, D, in the plane containing a point (x_o,y_o). Suppose that the first order partial derivatives of f at (x_o,y_o) exist. Let $\theta\in[0,2\pi)$. The directional derivative of f at (x_o,y_o) in the direction of the angle θ, denoted by $D_\theta f(x_o,y_o)$, is defined by

$$D_\theta f(x_o,y_o)=\frac{\partial f}{\partial x}(x_o,y_o)\cdot\cos\theta+\frac{\partial f}{\partial y}(x_o,y_o)\cdot\sin\theta.$$

Do the following problems

1. Let $f\colon D\to\mathbb{R}$ have partial derivatives at (x_o,y_o), for $(x_o,y_o)\in D$. Compute the directional derivatives: (a) $D_0 f(x_o,y_o)$, and (b) $D_{\pi/2}f(x_o,y_o)$.

2. Let $f(x,y)=x^2+y^2$ for all $(x,y)\in\mathbb{R}^2$. Compute the directional derivative $D_\theta f(2,1)$ when (a) $\theta=\pi/4$, and (b) $\theta=-\pi/4$.

3. Let $f(x,y)=3xy+y^2$ for all $(x,y)\in\mathbb{R}^2$. Compute the rate of change of f at $(2,3)$ in the direction of the vector $\overrightarrow{v}=3\widehat{i}-\widehat{j}$.

4. Let $f(x,y)=\dfrac{x+y}{1+x^2}$ for all $(x,y)\in\mathbb{R}^2$. Compute the rate of change of f at $(1,-2)$ in the direction of the vector $\overrightarrow{v}=3\widehat{i}-2\widehat{j}$.

5. The directional derivative of a function, f, of two variables, x and y, at $(2,1)$ in the direction towards the point $(1,3)$ is $-2/\sqrt{5}$, and the directional derivative at $(2,1)$ in the direction of towards the point $(5,5)$ is 1. Compute the first–order partial derivatives of f at $(2,1)$.