## Assignment #12

**Due on Monday, March 9, 2015**

**Read** Section 14.3, on *Local Linearity and the Differential,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

**Read** Section 14.4, on *Gradients and Directional Derivatives in the Plane,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

**Read** Section 14.6, on *The Chain Rule,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

**Background and Definitions**.
**The Chain Rule (Version I)**. Let $f\colon D \to \mathbb{R}$ be a real–valued function defined on some domain, $D$, in the $xy$–plane, and let $\overrightarrow{r}\colon I \to \mathbb{R}^2$, for some open interval $I$, denote a differentiable path with $\overrightarrow{r}(t) \in D$ for all $t \in I$. Suppose that the partial derivatives of $f$ exist and are continuous in $D$. Then, for any $t \in I$,

$$\frac{d}{dt}[f(\overrightarrow{r}(t))] = \frac{\partial f}{\partial x}(\overrightarrow{r}(t))\frac{dx}{dt} + \frac{\partial f}{\partial y}(\overrightarrow{r}(t))\frac{dy}{dt},$$

where $\overrightarrow{r}(t) = (x(t), y(t)$ for all $t \in I$.

**Do** the following problems

1. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ have continuous partial derivatives for all $(x, y) \in \mathbb{R}^2$, and $\overrightarrow{r}(t) = at\widehat{i} + bt\widehat{j}$, for all $t \in \mathbb{R}$, where $a$ and $b$ are given real numbers.

   Apply the Chain Rule to compute $\dfrac{d}{dt}[f(\overrightarrow{r}(t))]$.

2. A bug is moving on a two–dimensional plate, $D$, with temperature $u(x, y)$ for all $(x, y) \in D$. Assume that at $(x_o, y_o) \in D$,

   $$\frac{\partial u}{\partial x}(x_o, y_o) = -2 \quad \text{and} \quad \frac{\partial u}{\partial y}(x_o, y_o) = 1.$$

   Suppose the velocity of the bug at when it is at $(x_o, y_o)$ is given by the vector $v = 4\widehat{i} + 7\widehat{j}$. Compute the rate of change of temperature along the path of the but at the point $(x_o, y_o)$.

3. Apply the Chain Rule to obtain $\dfrac{dz}{dt}$, where $z = xy^2$ and $(x(t), y(t)) = (e^{-t}, \sin t)$ for all $t \in \mathbb{R}$.

4. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ have continuous partial derivatives for all $(x, y) \in \mathbb{R}$. Let $C$ denote the level curve $f(x, y) = c$, for some constant $c$. Let $(a, b)$ be a point on the curve $C$; so that $f(a, b) = c$. Assume that
$$\frac{\partial f}{\partial y}(a, b) \neq 0.$$
Use the Chain Rule to compute the slope of the line tangent to $C$ at the point $(a, b)$.

5. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ have continuous partial derivatives for all $(x, y) \in \mathbb{R}$. Suppose also that
$$f(tx, ty) = t^2 f(x, y), \qquad \text{for all } (x, y) \in \mathbb{R}^2 \text{ and all } t \in \mathbb{R}. \tag{1}$$
Verify that $f$ satisfies the equation
$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f.$$
*Suggestion:* Differentiate with respect to $t$ on both sides of (1); apply the Chain Rule on the left–hand side; and then make the substitution $t = 1$.