

## Assignment #9

Due on Friday, February 27, 2015

**Read** Section 12.4, on *Linear Functions*, in *Calculus: Multivariable*, by McCallum, Hughes–Hallett, Gleason, et al.

**Do** the following problems

1. Give the formula for computing a linear function,  $f$ , whose graph is the plane passing through the points  $(0, 0, 0)$ ,  $(0, 2, -1)$  and  $(-3, 0, 4)$ .

Sketch the plane.

2. Give the equation for the plane containing the line in the  $xy$ -plane where  $y = 1$ , and the line in the  $xz$ -plane where  $z = 2$ .

Sketch the plane.

3. A linear function  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  is given by the formula

$$f(x, y) = d + ax + by, \quad \text{for all } (x, y) \in \mathbb{R}^2,$$

where  $a$ ,  $b$  and  $d$  are real numbers.

Determine values for  $a$ ,  $b$  and  $d$  so that the graph of  $z = f(x, y)$  intersects the  $xz$ -plane in the line  $z = 3x + 4$  and intersects the  $yz$ -plane in the line  $z = y + 4$ .

4. In each of the following, sketch the graph of  $z = f(x, y)$  for the given linear function  $f$ .

(a)  $f(x, y) = 2 - x - 2y$ , for all  $(x, y) \in \mathbb{R}^2$ .

(b)  $f(x, y) = 4 + x - 2y$ , for all  $(x, y) \in \mathbb{R}^2$ .

5. A linear function  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  is given by the formula

$$f(x, y) = d + ax + by, \quad \text{for all } (x, y) \in \mathbb{R}^2,$$

where  $a$ ,  $b$  and  $d$  are real numbers such that  $b \neq 0$ .

(a) Verify that the contours of  $f$  are lines of slope  $-a/b$ .

(b) Verify that  $f(x + b, y - a) = f(x, y)$  for all  $(x, y) \in \mathbb{R}^2$ .

(c) Give an interpretation for the results in parts (a) and (b).