Review Problems for Exam 1

1. Sketch the curve C parametrized by
$$\begin{array}{rcl} x & = & \sin^2(t); \\ y & = & \cos^2(t), \end{array} \qquad \text{for } -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}.$$

2. A curve C is parametrized by the differentiable path given by
$$\overrightarrow{r}(t) = (3t^2, 2 + 5t), \qquad \text{for } t \in \mathbb{R}.$$
Sketch the curve C in the xy–plane. Describe the curve.

3. Sketch the curve C parametrized by
$$\begin{array}{rcl} x & = & 2 + 3\cos t; \\ y & = & 1 + \sin t, \end{array} \qquad \text{for } 0 \leqslant t \leqslant 2\pi.$$
Describe the curve.

4. Give a parametrization for the portion of the circle of radius 2 centered at $(1, 1)$ from the point $P(1, 3)$ to the point $Q(3, 1)$.

5. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ denote distinct points in the plane. Give a parametrization of the directed line segment $\overrightarrow{PQ}$.

6. Compute and sketch the flow of the vector field
$$\overrightarrow{F}(x, y) = -2x\widehat{i} + y\widehat{j}, \qquad \text{for } (x, y) \in \mathbb{R}^2.$$

7. Compute and sketch the flow of the vector field
$$\overrightarrow{F}(x, y) = -2x\widehat{i} - 2y\widehat{j}, \qquad \text{for } (x, y) \in \mathbb{R}^2.$$

8. Let C denote the unit circle in the xy–plane centered at the origin. Give the coordinates of the points on C at which the tangent line is parallel to the line $y = x$.

9. Give the linear approximation to the path $\overrightarrow{r}(t) = (t^3, 2 + t^2)$, for $t \in \mathbb{R}$, at the point $(1, 3)$.

10. Compute the arc length along the curve parametrized by
$$\begin{array}{rcl} x & = & r\cos t; \\ y & = & r\sin t, \end{array} \qquad \text{for } 0 \leqslant t \leqslant \theta,$$
where θ is a given positive real number.