Assignment #3

Due on Wednesday, February 1, 2017

Read Section 4.1 on *Solving Two–dimensional Linear Systems* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 2.2, on *The Geometry of Systems,* in Blanchard, Devaney and Hall.

Do the following problems

1. Find the solutions curves to the autonomous, linear system

$$\begin{cases} \dfrac{dx}{dt} &= x; \\ \dfrac{dy}{dt} &= y, \end{cases}$$

   and sketch the phase portrait.

   Give the solution subject to the initial condition $x(0) = 1$, $y(0) = 1$.

2. Find the solutions curves to the autonomous, linear system

$$\begin{cases} \dfrac{dx}{dt} &= x; \\ \dfrac{dy}{dt} &= 2y, \end{cases}$$

   and sketch the phase portrait.

   Give the solution subject to the initial condition $x(0) = 1$, $y(0) = 0$.

3. Find the solutions curves to the autonomous, linear system

$$\begin{cases} \dfrac{dx}{dt} &= x; \\ \dfrac{dy}{dt} &= -2y, \end{cases}$$

   and sketch the phase portrait.

   Give the solution subject to the initial condition $x(0) = 0$, $y(0) = 1$.

4. Consider the linear system

$$\begin{cases} \dfrac{dx}{dt} &= x + 1; \\ \dfrac{dy}{dt} &= 1 - y. \end{cases} \tag{1}$$

   (a) Make the change of variable $u = x + 1$ and $v = 1 - y$ and express the system in (1) as a system in terms of u and v:

$$\begin{cases} \dot{u} &= f(u, v); \\ \dot{v} &= g(u, v). \end{cases} \tag{2}$$

   (b) Find the solutions curves of the system in (2) and sketch the phase portrait in the uv–plane.

   (c) Use the information gained in part (b) to sketch the phase portrait of the system in (1) in the xy–plane.

5. Consider the linear system

$$\begin{cases} \dfrac{dx}{dt} &= 5x + 3y; \\ \dfrac{dy}{dt} &= -6x - 4y. \end{cases} \tag{3}$$

   (a) Make the change of variable $u = 2x + y$ and $v = x + y$ and express the system in (3) as a system in terms of u and v:

$$\begin{cases} \dot{u} &= f(u, v); \\ \dot{v} &= g(u, v). \end{cases} \tag{4}$$

   (b) Find the solutions curves of the system in (4) and sketch the phase portrait in the uv–plane.

   (c) Use the information gained in part (b) to sketch the phase portrait of the system in (3) in the xy–plane.