Assignment #6

Due on Monday, February 13, 2017

Read Section 4.1.5 on *Non–Diagonalizable Systems with No Real Eigenvalues* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 3.4 on *Complex Eigenvalues* in Blanchard, Devaney and Hall.

Do the following problems

1. For the following linear system, give the equations for the solution curves and sketch the phase portrait.

$$\begin{cases} \dot{x} &= -x + y; \\ \dot{y} &= -x - y. \end{cases}$$

2. Consider the system

$$\begin{cases} \dfrac{dx}{dt} &= y; \\ \dfrac{dy}{dt} &= -\omega^2 x, \end{cases} \tag{1}$$

where ω is a positive constant.

   (a) Use the change of variables

$$u = \frac{1}{\omega} y$$

$$v = x$$

   to turn system (1) into a system in the u and v variables.

   (b) Solve the system in part (b) and use it to construct solutions of the system in (1).

   (c) Sketch the phase–portrait of the system in (1).

3. Sketch the phase portrait of the system

$$\begin{cases} \dfrac{dx}{dt} &= -x + 4y; \\ \dfrac{dy}{dt} &= -2x + 3y. \end{cases}$$

4. Turn the second order equation

$$x'' + x = 0 \tag{2}$$

into a two–dimensional linear system; construct solutions of the system; and use the solutions of the system to construct solutions of (2).

Give a solution of (2) subject to the initial conditions $x(0) = 0$, $x'(0) = 1$.

5. Turn the second order equation

$$x'' - x' - 2x = 0 \tag{3}$$

into a two–dimensional linear system; construct a solution of the system; and use the solution of the system to construct solutions of (3).

Give a solution of (3) subject to the initial conditions $x(0) = 1$, $x'(0) = 0$.