

Assignment #3

Due on Wednesday, January 31, 2018

Read Section 4.1 on *Solving Two-dimensional Linear Systems* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 2.2, on *The Geometry of Systems*, in Blanchard, Devaney and Hall.

Do the following problems

1. Find the solutions curves to the autonomous, linear system

$$\begin{cases} \frac{dx}{dt} = x; \\ \frac{dy}{dt} = y, \end{cases}$$

and sketch the phase portrait.

Give the solution subject to the initial condition $x(0) = 1$, $y(0) = 1$.

2. Find the solutions curves to the autonomous, linear system

$$\begin{cases} \frac{dx}{dt} = x; \\ \frac{dy}{dt} = 2y, \end{cases}$$

and sketch the phase portrait.

Give the solution subject to the initial condition $x(0) = 1$, $y(0) = 0$.

3. Find the solutions curves to the autonomous, linear system

$$\begin{cases} \frac{dx}{dt} = x; \\ \frac{dy}{dt} = -2y, \end{cases}$$

and sketch the phase portrait.

Give the solution subject to the initial condition $x(0) = 0$, $y(0) = 1$.

4. Consider the linear system

$$\begin{cases} \frac{dx}{dt} = x + 1; \\ \frac{dy}{dt} = 1 - y. \end{cases} \quad (1)$$

(a) Make the change of variable $u = x + 1$ and $v = 1 - y$ and express the system in (1) as a system in terms of u and v :

$$\begin{cases} \dot{u} = f(u, v); \\ \dot{v} = g(u, v). \end{cases} \quad (2)$$

(b) Find the solutions curves of the system in (2) and sketch the phase portrait in the uv -plane.

(c) Use the information gained in part (b) to sketch the phase portrait of the system in (1) in the xy -plane.

5. Consider the linear system

$$\begin{cases} \frac{dx}{dt} = 5x + 3y; \\ \frac{dy}{dt} = -6x - 4y. \end{cases} \quad (3)$$

(a) Make the change of variable $u = 2x + y$ and $v = x + y$ and express the system in (3) as a system in terms of u and v :

$$\begin{cases} \dot{u} = f(u, v); \\ \dot{v} = g(u, v). \end{cases} \quad (4)$$

(b) Find the solutions curves of the system in (4) and sketch the phase portrait in the uv -plane.

(c) Use the information gained in part (b) to sketch the phase portrait of the system in (3) in the xy -plane.