Assignment #9

Due on Monday, March 5, 2017

Read Section 4.2.1 on *Fundamental Matrices* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`.

Read Section 4.2.2 on *Existence and Uniqueness* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`.

Do the following problems

1. Let $J = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$. Verify that the corresponding fundamental matrix, E_J, commutes with J.

2. Let $J = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$. Verify that the corresponding fundamental matrix, E_J, commutes with J.

3. Let $J = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$. Verify that the corresponding fundamental matrix, E_J, commutes with J.

4. Let A denote a 2×2 matrix with real coefficients, and let E_A denote the fundamental matrix fore the system
$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}.$$
Show that A and E_A commute.

5. Let E_A denote the fundamental matrix of the system
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$
and put $Y(t) = [E_A(t)]^{-1}$, for all $t \in \mathbb{R}$. Show that Y is the fundamental matrix of the system
$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = -A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$