Topics for Final Exam

1. **Modeling via Differential Equations**
   - 1.1 Conservation principles and compartment models
   - 1.2 Nondimesionalization
   - 1.3 Qualitative analysis of models
2. **Types of Differential Equations**
   - 2.1 Order
   - 2.2 Linear versus nonlinear
   - 2.3 Systems
   - 2.4 Autonomous systems
3. **Solving Linear Systems of Differential Equations**
   - 3.1 General solutions
   - 3.2 Initial conditions
   - 3.3 Solving two–dimensional linear systems with constant coefficients
     - 3.1 Homogeneous and nonhomogeneous systems
     - 3.2 Diagonalizable systems
     - 3.3 Non–diagonalizable systems
   - 3.4 One–dimensional Systems
     - 4.1 Separation of variables
     - 4.2 Integrating factors
   - 3.5 Second order equations
4. **Phase–Plane Analysis of Linear Systems**
   - 4.1 Solution curves (trajectories, orbits)
   - 4.2 Phase portrait: nullclines, equilibrium points, stability
5. **Analysis of Linear Systems**
   - 5.1 Construction of solutions
   - 5.2 Linearly independent solutions
   - 5.3 The fundamental matrix
   - 5.4 Existence and uniqueness for linear, initial value problems

6. **Applications to Second Order Linear Differential Equations**

    6.1 Construction of solutions of linear second order equations.

    6.2 Linearly independent solutions.

    6.3 Structure of the solutions space for linear, second-order, homogeneous differential equations (Problem 3 in Assignment #10)

    6.4 Existence and uniqueness for the initial value problem.

7. **Analysis of General Systems**

    7.1 Local Existence and Uniqueness Theorem.

    7.2 Maximal interval of existence.

    7.3 Global existence.

8. **Phase–Plane Analysis of General Two–Dimensional Systems**

    8.1 Nullclines and equilibrium points

    8.2 Stability

    8.3 Classification of equilibrium points

        3.1 Stable: Center, sink, spiral sink

        3.2 Unstable: Saddle point, source, spiral source

    8.4 Principle of Linearized Stability

    8.5 Cycles and periodic solutions

9. **Qualitative Analysis of a Single Differential Equation**

    9.1 Analysis of first–order equations

        1.1 Equilibrium points and stability

        1.2 Principle of linearized stability

        1.3 Long–term behavior of solutions

    9.2 Analysis of second order equations

        2.1 Phase plane analysis

        2.2 Oscillations

10. **Special Types of Systems**: Conservative systems

Relevant sections in the online class notes: 2.1, 2.2, 3.1, 4.1, 4.2, 5.1, 5.2 5.3, 6.1, 6.2, 6.3 and 6.4.

Relevant sections in text: 1.1, 1.2, 1.5, 1.6, 1.8, 1.9, 2.1, 2.2, 2.6, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 5.1, 5.2 and 5.3

Important concepts: Solution curves (trajectories, orbits); phase portrait; general solutions; diagonalizability; Jordan canonical forms; linearly independent functions; fundamental matrix; local existence and uniqueness; maximal interval of existence; global existence and uniqueness; nullclines; equilibrium points; stability of equilibrium points; linearization; Principle of Linearized Stability; asymptotic stability; neutral stability; source; sink; saddle point; center; spiral sink; spiral source; cycles; oscillations; conserved quantities.

Important skills:

- know how to apply conservation principles to obtain differential equations models;
- know how to solve linear first order equations by separating variables, or using integrating factors;
- know how to turn a second order equation into a two–dimensional system;
- know how to construct solutions of homogeneous, two–dimensional systems with constant coefficients;
- know how to compute fundamental matrices for homogenous, autonomous linear systems;
- know how to construct solutions of second order linear equations with constant coefficients;
- know how to apply the existence and uniqueness theorems;
- know how to nondimensionalize systems involving parameters;
- know how to use nullclines and the principle of linearized stability to sketch the phase portrait of general, two–dimensional, autonomous systems;
- know how to classify equilibrium points of general, two–dimensional, autonomous systems;
- know how to apply the principle of linearized stability for a single first order equation;
- know how to determine the long–term behavior of solutions of a single, autonomous, first–order equation;
- know how to apply phase–plane analysis to a single, second–order equation;
- Know how to find conserved quantities for conservative systems.