Assignment #1

Due on Wednesday, January 30, 2019

Read Chapter 1 in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Chapter 2, *Introduction: An Example from Epidemiology,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Background and Definitions.
In Section 2.1 of the class lecture notes, we derived the Kermack–McKendrick SIR model for the spread of an infections disease in a population,

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI; \\ \dfrac{dI}{dt} = \beta SI - \gamma I; \\ \dfrac{dR}{dt} = \gamma I. \end{cases} \tag{1}$$

The quantity $S(t)$ denotes the number of individuals in the population that are susceptible to getting the disease, $I(t)$ is the number of individuals that have contracted the disease and can infect individuals from the susceptible class, and $R(t)$ is the number of individuals in the population that have recovered and are immune to the disease. The positive parameters β and γ are called the infection rate and recovery rate, respectively.

Do the following problems

1. Give units for the parameters β and γ in the SIR system in (1).

2. Let $N(t) = S(t) + I(t) + R(t)$ for all t. Use the equations in (1) to derive the differential equation
$$\frac{dN}{dt} = 0.$$
Deduce that $N(t)$ must be a constant function.

3. Let $S_o = S(t)$, the initial number of susceptible individuals in the population and put
$$R_o = \frac{\beta S_o}{\gamma}. \tag{2}$$
Give the units for R_o.

   The constant R_o is called the reproduction number.

4. Assume that at time $t = 0$, there are is only one infectious individual in the population and no one in the population has acquired immunity. Let N denote the total number of individuals in the population.

   (a) Compute S_o in terms of N.

   (b) Give the reproduction number, R_o, in (2) in this situation.

5. Let R_o be as computed in Problem 4.

   (a) Assume that $R_o > 1$, and determine the sign of $I'(0)$. What do you conclude in this case? Explain the reasoning leading to your conclusion.

   (b) Assume that $R_o < 1$, and determine the sign of $I'(0)$. What do you conclude in this case? Explain the reasoning leading to your conclusion.