Assignment #12

Due on Wednesday, March 27, 2019

Read Chapter 5, on *Linear Vector Fields in Two Dimensions,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Let A be the 2×2 matrix given by $A = \begin{pmatrix} -1 & 1 \\ 5 & -1 \end{pmatrix}$. Let v and w denote the column vectors $\mathrm{v} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ and $\mathrm{w} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Compute $A\mathrm{v}$ and $A\mathrm{w}$.

2. Let A, v and w be as in Problem 1. Compute the vector $2\mathrm{v} - 3\mathrm{w}$ and compute the product $A(2\mathrm{v} - 3\mathrm{w})$. Verify that
$$A(2\mathrm{v} - 3\mathrm{w}) = 2A\mathrm{v} - 3A\mathrm{w}.$$

3. Find a condition on the scalars a, b, c and d so that the columns of the matrix
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
are not scalar multiples of each other; that is the column vectors of A do not lie on the same line.

   Suggestion: Consider the cases $a = 0$ and $a \neq 0$ separately.

4. Let the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfy the condition you discovered in Problem 3. Show that the matrix equation $A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ has only one solution; namely, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

5. Let A denote the matrix in Problem 1. Let v_1 denote the first column of A and v_2 denote the second column of A. Find scalars c_1 and c_2 for which
$$c_1\mathrm{v}_1 + c_2\mathrm{v}_2 = \begin{pmatrix} 4 \\ 7 \end{pmatrix}.$$