Assignment #19

Due on Monday, April 29, 2019

Read Section 6.2, on *Linear Approximations,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.3, on *Linear Approximations and Partial Derivatives,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Compute the first partial derivatives of the function f given by

$$f(x,y) = \frac{x}{x^2+y^2}, \quad \text{for } (x,y) \neq (0,0).$$

2. Compute the first partial derivatives of the function f given by

$$f(x,y) = e^{-x}\sin y, \quad \text{for all } (x,y) \in \mathbb{R}^2.$$

3. Find a function f of the variables x and y satisfying

$$\begin{aligned} \frac{\partial f}{\partial x}(x,y) &= y + 2x; \\ \frac{\partial f}{\partial y}(x,y) &= x, \end{aligned}$$

   for all $(x,y) \in \mathbb{R}^2$.

4. Let f be as in Problem 2.

   Compute the second partial derivatives of f:

$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y \partial x} \quad \text{and} \quad \frac{\partial^2 f}{\partial y^2}.$$

5. Let $f(x,y) = e^{-x}\cos y$ for all $(x,y) \in \mathbb{R}^2$.

   Verify that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$