Solutions to Assignment #3

1. Give a parametrization of the portion of the graph of $y = \sqrt{x}$ from the point $(1, 1)$ the point $(16, 4)$.

   Sketch the curve.

   Solution: A parametrization is given by

   $$(x(t), y(t)) = (t, \sqrt{t}), \qquad \text{for } 1 \leqslant t \leqslant 16.$$

   A sketch of the curve is shown in Figure 1. □

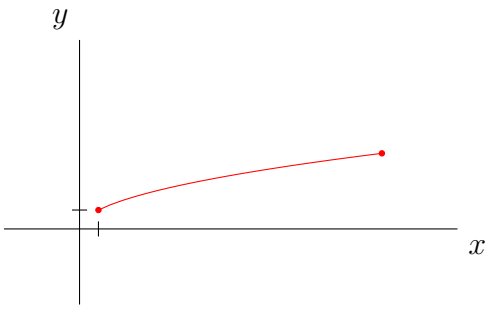

Figure 1: Sketch of Curve in Problem 1

2. Give a parametrization of the portion of the ellipse given by the graph of the

   $$x^2 + 4y^2 = 4 \tag{1}$$

   in the first quadrant.

   Sketch the curve.

   Solution: Divide both sides of the equation in (1) by 4 to get

   $$\frac{x^2}{2} + y^2 = 1. \tag{2}$$

   It follows from (2) that a parametrization of the portion of the ellipse in (2) is given by

   $$\begin{cases} x(t) &= 2\cos t; \\ y(t) &= \sin t, \end{cases} \qquad \text{for } 0 \leqslant t \leqslant \frac{\pi}{2}. \tag{3}$$

   A sketch of the curve parametrized by (3) is shown in Figure 2. □

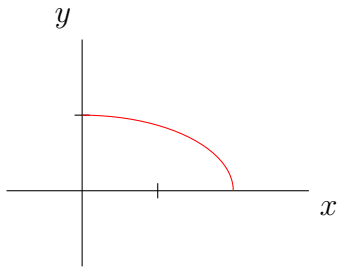

Figure 2: Sketch of Curve in Problem 2

3. Give a parametrization of a circular arc from the point $P(0,0)$ to the point $Q(10,0)$ on a circle of radius 5.

   Sketch the curve.

   Solution: There are two possible curves along the circle that connect the point $P(0,0)$ to the point $Q(10,0)$.

   The circle in question is centered at $(5,0)$ and has radius 5; thus, it has equation

   $$(x-5)^2 + y^2 = 25. \tag{4}$$

   It follows from (4) that a parametrization of the arc is given by

   $$\begin{cases} x(t) &= 5 + 5\sin t; \\ y(t) &= 5\cos t, \end{cases} \qquad \text{for } -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}. \tag{5}$$

   A sketch of the curve parametrized by (5) is shown in Figure 3.

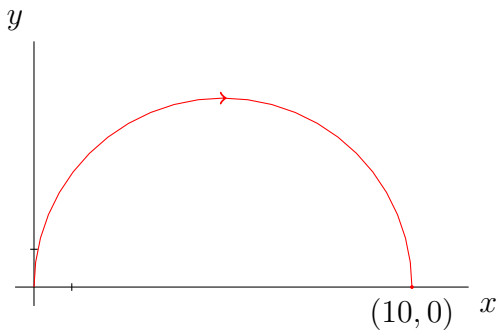

Figure 3: Sketch of Curve in Problem 3

The other arc along the circle in (4) connecting the point $P(0,0)$ to the point $Q(10,0)$ is the portion of the circle that lies below the x–axis. The parametrization of that arc if given by

$$\begin{cases} x(t) &= 5+5\sin t; \\ y(t) &= -5\cos t, \end{cases} \qquad \text{for } -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}. \tag{6}$$

A sketch of the curve parametrized by (6) is shown in Figure 4. □

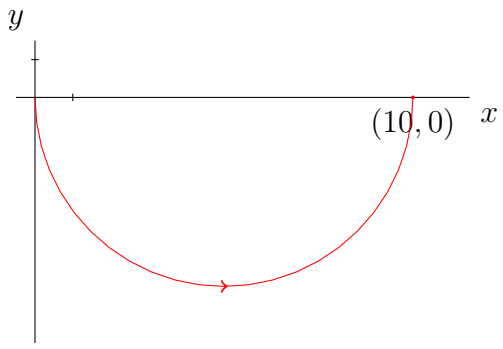

Figure 4: Sketch of Curve in Problem 3

4. Give a parametrization of the straight line segment from the point $P(2,5)$ to the point $Q(12,9)$.

   Sketch the curve.

   Solution: A parametrization of the directed line segment $\overrightarrow{PQ}$ is

$$\begin{cases} x(t) &= 2+10t; \\ y(t) &= 5+4t, \end{cases} \qquad \text{for } 0 \leqslant t \leqslant 1. \tag{7}$$

   A sketch of $\overrightarrow{PQ}$ is shown in Figure 5. □

5. Give a parametrization of the straight line through the point $P(2,1)$ that is parallel to the line $y = 2x$.

   Sketch the curve.

   Solution: A parametrization of the line is

$$\begin{cases} x(t) &= 2+t; \\ y(t) &= 1+2t, \end{cases} \qquad \text{for } t \in \mathbb{R}, \tag{8}$$

   since the slope of the line is 2. A sketch of the line is shown in Figure 6. □

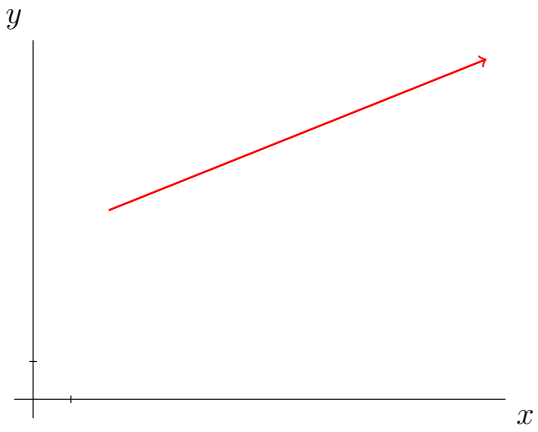

Figure 5: Sketch of Directed Line Segment in Problem 4

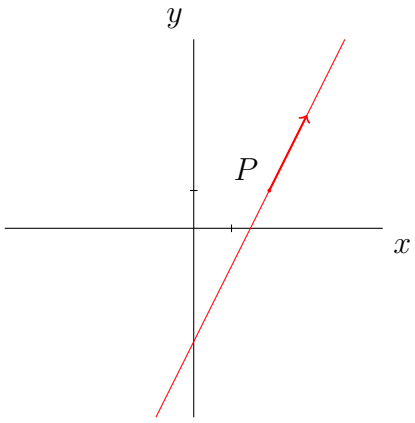

Figure 6: Sketch of Directed Line Segment in Problem 5