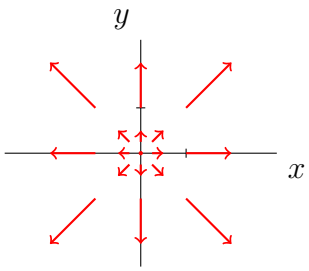
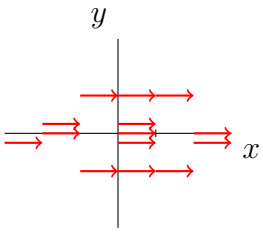
Solutions to Assignment #7

1. Give a formula defining the vector field $F(x,y) = f(x,y)\hat{i} + g(x,y)\hat{j}$, where f and g are real valued functions defined on the plane, whose picture is shown below.



Solution: $F(x,y) = x\hat{i} + y\hat{j}$, for all $(x,y) \in \mathbb{R}^2$. □

2. Give a formula defining the vector field $\overrightarrow{F}(x,y) = f(x,y)\overrightarrow{i} + g(x,y)\widehat{j}$, where f and g are real valued functions defined on the plane, whose picture is shown below.



Solution: $F(x,y) = \hat{i}$, for all $(x,y) \in \mathbb{R}^2$. □

3. Sketch the vector field $F(x,y) = 2\hat{i} + 3\hat{j}$,

Solution: Some of the vectors in the vector field are sketched in Figure 1. □

4. Sketch the vector field $F(x,y) = y\hat{j}$,

Solution: Some of the vectors in the vector field are sketched in Figure 2. □

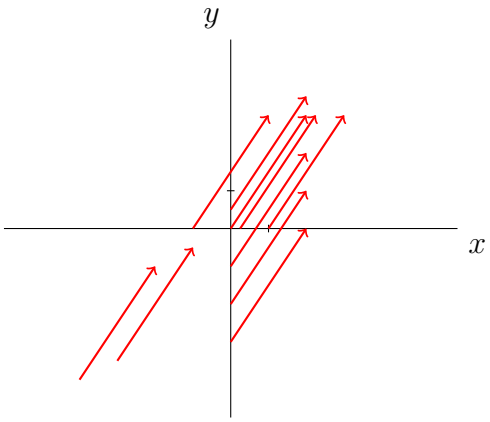


Figure 1: Sketch of Vector Field in Problem 3

5. Sketch the vector field $F(v) = \dfrac{1}{\|v\|}\, v$, where $v = x\hat{i} + y\hat{j} \neq (0,0)$.

   Solution: At each point in (x, y) in the xy–plane, $F(x, y)$ is a unit vector placed at (x, y) in the direction of $x\hat{i} + y\hat{j}$; that is, the vector points radially away from the origin. So of these vectors are sketched in Figure 3. □

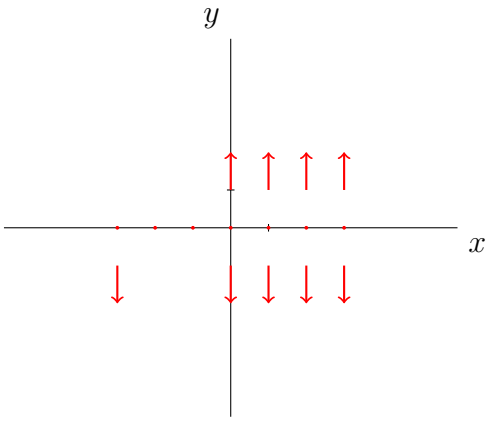


Figure 2: Sketch of Vector Field in Problem 4

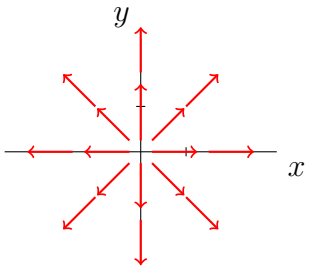


Figure 3: Sketch of Vector Field in Problem 5