Assignment #20

Due on Wednesday, May 6, 2020

Read Section 9.1 on *Point Estimation* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 9.2 on *Estimating the Mean* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.8 on *The Sample Mean* in DeGroot and Schervish.

Do the following problems

1. Let X denote a random variable with mean μ and variance σ^2. Use Chebyshev's inequality to show that
$$\Pr(|X-\mu| \geqslant k\sigma) \leqslant \frac{1}{k^2},$$
for all $k > 0$.

2. Suppose that a factory produces a number X of items in a week, where X can be modeled by a random variable with mean 50. Suppose also that the variance for a week's production is known to be 25. What can be said about the probability that this week's production will be between 40 and 60?

3. How large a random sample must be taken from a given distribution in order for the probability to be at least 0.99 that the sample mean will be within 2 standard deviations of the mean of the distribution?

4. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size n from a distribution for which the mean is 6.5 and the variance is 4. Determine how large the value of n must be in order for the following relation to be satisfied:
$$\Pr(6 \leqslant \overline{X}_n \leqslant 7) \geqslant 0.8.$$

5. Suppose that 30% of the items in a large manufactured lot are of poor quality. Suppose also that a random sample of n items is to be taken from the lot, and let Q_n denote the proportion of the items in the sample that are of poor quality. Use the Chebyshev inequality to find the value of n such that
$$\Pr(0.2 \leqslant Q_n \leqslant 0.4) \geqslant 0.75.$$