Assignment #6

Due on Monday, February 17, 2020

Read Section 3.5 on *Independent Events* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Sections 3.6 on *Conditional Probability* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 2.1 on *The Definition of Conditional Probability* in DeGroot and Schervish.

Read Section 2.2 on *Independent Events* in DeGroot and Schervish.

Do the following problems

1. Toss a balanced die twice in a row. Let E_1 denote the event that the first toss yields either a 1, or a 2, or a 3; E_2 the event that the first toss yields a 3, or a 4, or a 5; and E_3 the event that the sum of the outcomes of the two tosses is 9.

   (a) Verify that
   $$\Pr(E_1 \cap E_2 \cap E_3) = \Pr(E_1) \cdot \Pr(E_2) \cdot \Pr(E_3).$$

   (b) Show that E_1, E_2 and E_3 are not pairwise independent.

2. An experiment consists of drawing 2 balls at random from a bin containing 3 red balls and 2 green balls (the balls are not replaced between draws). Let A be the event that both balls are red, B the event that both balls are green, and C the event that the first ball is red.

   (a) Compute the probabilities of A, B, and C.

   (b) Compute $\Pr(A \mid B)$ and $P(B \mid A)$.

   (c) Are the events A and B independent? Justify your answer.

3. A box contains three coins: two regular coins and one fake, two–headed coin.

   (a) You pick a coin at random and toss it. What is the probability that it lands heads up?

   (b) You pick a coin at random and toss it, and get heads. What is the probability that it is the two–headed coin?

4. A researcher is studying the prevalence of three health risk factors, denoted A, B and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only that risk factor and not the others. For any two of the three factors, the probability is 0.12 a woman has exactly those two risk factors, but not the other. Assume that the probability that a woman has all three risk factors, given that she has A and B, is 1/3. Compute the probability that a woman has none of the three risk factors, given that she does not have risk factor A.

5. **The Monty Hall Problem**. In a game show there are three curtains. Behind one curtain is a nice prize while behind the other two there are worthless prizes. A contestant selects one curtain at random, and then Monte Hall (the game show host) opens one of the other two curtains to reveal a worthless prize. Hall then expresses the willingness to trade the curtain that the contestant has selected for the other curtain that has not been opened. Should the contestant switch curtains or stick with the one that she has? If she sticks with the one she has then the probability of winning the prize is 1/3. Hence, to answer this question, you must determine the probability that she wins the prize given that she switches.