Review Problems for Exam 2

1. A bowl contains 5 chips of the same size and shape. Two chips are red and the other three are blue. Draw three chips from the bowl at random, without replacement. Let X denote the number of blue chips in a drawing.

   (a) Give the pmf of X.

   (b) Compute $\Pr(X > 1)$.

   (c) Compute $E(X)$.

2. Let X have pmf given by $p_X(x) = \dfrac{1}{3}$, for $x = 1, 2, 3$, and $p(x) = 0$ elsewhere. Give the pmf of $Y = 2X + 1$.

3. A player simultaneously rolls a fair die and flips a fair coin. If the coin lands heads, she wins twice the value of the die roll (in dollars). If it lands tails, she wins half. Compute the expected earnings of the player.

4. A *mode* of a distribution of a random variable X is a value of x that maximizes the pdf, or the pmf of X. If there is only one such value, it is called *the mode of the distribution.* Find the mode for each of the following distributions:

   (a) $p(x) = \left(\dfrac{1}{2}\right)^x$, for $x = 1, 2, 3, \ldots$, and $p(x) = 0$, elsewhere.

   (b) $f_X(x) = \begin{cases} 12x^2(1 - x), & \text{if } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$

5. Let X have pdf
$$f_X(x) = \begin{cases} 2x, & \text{if } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

   Compute the probability that X is at least $3/4$, given that X is at least $1/2$.

6. Let X have pdf

$$f_X(x) = \begin{cases} x^2/9, & \text{if } 0 < x < 3; \\ 0, & \text{elsewhere.} \end{cases}$$

Find the pdf of $Y = X^3$.

7. Divide a segment at random into two parts. Find the probability that the largest segment is at least three times the shorter.

8. Assume that X and Y are independent, discrete random variables.

Show that $E(XY) = E(X)E(Y)$.

9. Assume that $X \sim \text{Uniform}(0, 1)$ and define $Y = -\ln X$.

   (a) Compute the cdf of Y.
   (b) Compute the pdf of Y
   (c) Compute $\Pr(Y > 1)$.
   (d) Compute $E(Y)$ and $\text{Var}(Y)$

10. A box contains a certain number of balls of various colors. Assume that 10% of the balls are red. If 20 balls are selected from the box at random, with replacement, what is the probability that more than 3 red balls will be obtained in the sample?

11. Let X denote a continuous random variable with pdf

$$f_X(x) = \begin{cases} \dfrac{x}{8}, & \text{if } 0 < x < 4; \\ \\ 0, & \text{elsewhere.} \end{cases}$$

Define Y to be the integer that is closest to X.

   (a) Explain why Y is a discrete random variable and give possible values for Y.
   (b) Compute the pmf of Y.

(c) Compute $E(Y)$ and $\text{Var}(Y)$.

12. Assume that X has a uniform distribution on the subset of the integers given by
$$\{1, 2, 3, \ldots, 47\}$$

   (a) Compute the probability of the event that X is even.

   (b) Compute the expected value of X.