Review Problems for Final Exam

1. Three cards are in a bag. One card is red on both sides. Another card is white on both sides. The third card is red on one side and white on the other side. A card is picked at random and placed on a table. Compute the probability that if a given color is shown on top, the color on the other side is the same as that of the top.

2. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and a number b of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Determine the value of b.

3. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

4. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

5. Suppose that $0 < \rho < 1$ and let $p(k) = C\rho^k$, for $k = 0, 1, 2, 3, \ldots$, and some constant $C > 0$.

   (a) Find the value of C so that p is the probability mass function (pmf) for a random variable.

   (b) Let X denote a discrete random variable with pmf p with the value of C found in part (a). Compute $\Pr(X > 1)$.

   (c) Let X denote a discrete random variable with pmf p with the value of C found in part (a). Compute compute the mgf of X.

   (d) Let X denote a discrete random variable with pmf p with the value of C found in part (a). Use the mgf of X computed in part (c) to compute the expected value and variance of X.

6. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x,y) = \frac{x+y}{8}, \qquad \text{for } 0 < x < 2 \text{ and } 0 < y < 2;$$

   and 0 elsewhere.

   What is the probability that the device fails during its first hour of operation?

7. Let $M(t)$ denote the number of mutations in a bacterial colony that occur during the interval $[0,t]$, assuming that $M(0) = 0$. Suppose that $M(t)$ has a Poisson(λt) distribution, where $\lambda > 0$ is a positive parameter.

   (a) Give an interpretation for λ.

   (b) Compute the probability that no mutations occur in the interval $[0,t]$.

   (c) Let T_1 denote the time that the first mutation occurs. Find the distribution of T_1.

8. A computer manufacturing company conducts acceptance sampling for incoming computer chips. After receiving a huge shipment of computer chips, the company randomly selects 800 chips. If three or fewer nonconforming chips are found, the entire lot is accepted without inspecting the remaining chips in the lot. If four or more chips are nonconforming, every chip in the entire lot is carefully inspected at the supplier's expense. Assume that the true proportion of nonconforming computer chips being supplied is 0.001. Estimate the probability the lot will be accepted.

9. A company manufactures a brand of light bulb with a lifetime in months that is normally distributed with mean 3 and variance 1 . A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have independent lifetimes. What is the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 40 months with probability at least 0.9772?

10. A random walk on the integer points on the x–axis begins at $x = 0$. At each step, the random walker is equally likely to move one unit to the left or to the right. Furthermore, the choice to move to the left or to the right is independent of the choices made in previous steps. Use the central limit theorem to estimate the probability that the random walker will be 10 units or more away from the origin after 100 steps.