

Topics for Final Exam**1. Probability Spaces**

- 1.1 Sample spaces
- 1.2 σ -Fields
- 1.3 Probability functions
- 1.4 Independent events
- 1.5 Conditional probability

2. Random Variables

- 2.1 Continuous and discrete random variables
- 2.2 Cumulative distribution function (cdf)
- 2.3 Probability density function (pdf) and probability mass function (pmf)

3. Expectations of Random Variables

- 3.1 Expected Value a random variable
- 3.2 Expected value of functions of random variables
- 3.3 Moments and moment generating function
- 3.4 Variance of a random variable

4. Joint Distributions

- 4.1 Joint distribution of two random variables
- 4.2 Marginal distributions
- 4.3 Independent random variables

5. Limiting Distributions

- 5.1 The Poisson distribution as a limit of binomial distributions.
- 5.2 Convergence in distribution
- 5.3 The mgf convergence theorem

6. The Central Limit Theorem

6.1 Statements of the central limit theorem

6.2 Applications of the Central Limit Theorem

7. The Weak Law of Large Numbers

7.1 Markov and Chebyshev inequalities

7.2 Convergence in probability

7.3 The weak law of large numbers

Relevant chapters and sections in the lecture notes: Chapters 2, 3, 4, 5, 6, 7 and 8; Sections 9.1 and 9.2.

Relevant sections in the text: 1.4, 1.5, 1.6, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 3.4, 3.5, 3.7, 3.8, 3.9, 4.1, 4.2, 4.3, 4.4, 4.6, 5.2, 5.3, 5.4, 5.6, 6.2, 6.3, 6.4, 8.1, 8.2, and 8.5.

Important Concepts

Sample space; σ -field; probability function; probability space; random variable; continuous and discrete random variables; cumulative distribution function (cdf); probability mass function (pmf); probability density function (pdf); expectation; moments; moment generating function (mgf); variance; mgf uniqueness theorem; joint distributions; marginal distributions; independent random variable; convergence in distribution; limiting distribution; the mgf convergence theorem; the central limit theorem; convergence in probability; the weak law of large numbers.

Some Special Distributions.

Discrete random variables: discrete uniform, Bernoulli, binomial, geometric, hypergeometric, Poisson.

Continuous random variables: Uniform, exponential, normal, chi-square and gamma.

Important skills

Know how to compute probabilities of events and apply the properties of probability; know how to compute conditional probabilities and apply the properties of conditional probability; know how to compute the cdf and the pdf (or pmf) of random variables; know how to compute expectations, moments, variance and moment generating functions of random variables; know how to compute the joint cdf and the joint pdf (or pmf) of two or more random variables; know how to compute marginal distributions; know how to compute probabilities based on joint distributions; know how to compute expectations of functions of random variables; know how to use independence; know how to apply the mgf uniqueness theorem; know how to apply the mgf convergence theorem; know how to apply the central limit theorem; know how to apply Chebyshev's inequality.