

Special Distributions

(I) Discrete Distributions

(1) *The Bernoulli Distribution*

$X \sim \text{Bernoulli}(p)$ for $0 < p < 1$

pmf: $p_X(x) = p^x(1-p)^{1-x}$ for $x = 0, 1$

Expected value: $E(X) = p$

Variance: $\text{Var}(X) = p(1-p)$

mgf: $\psi_X(t) = pe^t + 1 - p$ for $t \in \mathbf{R}$

(2) *The Binomial Distribution*

$X \sim \text{Binomial}(n, p)$ for $0 < p < 1$, $n = 2, 3, 4, \dots$

pmf: $p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \dots, n$

Expected value: $E(X) = np$

Variance: $\text{Var}(X) = np(1-p)$

mgf: $\psi_X(t) = (pe^t + 1 - p)^n$ for $t \in \mathbf{R}$

(3) *The Geometric Distribution*

$X \sim \text{Geometric}(p)$ for $0 < p < 1$

pmf: $p_X(x) = p(1-p)^{x-1}$ for $x = 1, 2, 3, \dots$

Expected value: $E(X) = \frac{1}{p}$

Variance: $\text{Var}(X) = \frac{1-p}{p^2}$

mgf: $\psi_X(t) = \frac{p}{e^{-t} + p - 1}$ for $t < \ln\left(\frac{1}{1-p}\right)$

(4) *The Discrete Uniform Distribution*

$X \sim \text{Discrete Uniform}(N)$ for $N = 1, 2, 3, \dots$

pmf: $p_X(x) = \frac{1}{N}$ for $x = 1, 2, \dots, N$

Expected value: $E(X) = (N+1)/2$

Variance: $\text{Var}(X) = (N+1)(N-1)/12$

mgf: $\psi_X(t) = \frac{1}{N} \sum_{i=1}^N e^{it}$ for $t \in \mathbf{R}$

(5) *The Hypergeometric Distribution*

$X \sim \text{Hypergeometric}(N, M, K)$ for $N, M, K > 0$ with $K < M$ and $M < N$.

pmf:
$$p_X(x) = \frac{\binom{M}{x} \binom{N-M}{K-x}}{\binom{N}{K}} \text{ for } x = 0, 1, 2, \dots, K$$

Expected value:
$$E(X) = \frac{KM}{N}$$

Variance:
$$\text{Var}(X) = \frac{KM}{N} \frac{(N-M)(N-K)}{N(N-1)}$$

(6) *The Poisson Distribution*

$X \sim \text{Poisson}(\lambda)$ for $\lambda > 0$

pmf:
$$p_X(x) = \frac{\lambda^x}{x!} e^{-\lambda} \text{ for } x = 0, 1, 2, 3, \dots$$

Expected value:
$$E(X) = \lambda$$

Variance:
$$\text{Var}(X) = \lambda$$

mgf:
$$\psi_X(t) = e^{\lambda(e^t - 1)} \text{ for } t \in \mathbf{R}$$

(II) **Continuous Distributions**(1) *The Uniform Distribution*

$X \sim \text{Uniform}(a, b)$ for $a < b$

pdf:
$$f_X(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b \text{ and } 0 \text{ elsewhere}$$

Expected value:
$$E(X) = \frac{a+b}{2}$$

Variance:
$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

mgf:
$$\psi_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t} \text{ for } t \neq 0 \text{ and } \psi_X(0) = 1$$

(2) *The Exponential Distribution*

$X \sim \text{Exponential}(\beta)$ for $\beta > 0$

pdf:
$$f_X(x) = \frac{1}{\beta} e^{-x/\beta} \text{ for } 0 \leq x < \infty \text{ and } 0 \text{ elsewhere}$$

Expected value:
$$E(X) = \beta$$

Variance:
$$\text{Var}(X) = \beta^2$$

mgf:
$$\psi_X(t) = \frac{1}{1 - \beta t} \text{ for } t < \frac{1}{\beta}$$

(3) *The Normal Distribution*

$X \sim \text{Normal}(\mu, \sigma^2)$ for $-\infty < \mu < \infty$ and $\sigma > 0$.

pdf:
$$f_X(x) = \frac{1}{\sqrt{2\pi} \sigma} \cdot e^{-(x-\mu)^2/(2\sigma^2)} \quad \text{for } -\infty < x < +\infty$$

Expected value: $E(X) = \mu$

Variance: $\text{Var}(X) = \sigma^2$

mgf: $\psi_X(t) = e^{\mu t + \sigma^2 t^2/2}$ for $t \in \mathbf{R}$

(4) *The Gamma Distribution*

$X \sim \text{Gamma}(\alpha, \beta)$, for $\alpha, \beta > 0$

pdf:
$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \quad \text{for } 0 < x < \infty$$

and zero elsewhere,

where Γ is the *gamma function* defined by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad \text{for all real values of } z \text{ except } 0, -1, -2, -3, \dots$$

Expected value: $E(X) = \alpha\beta$

Variance: $\text{Var}(X) = \alpha\beta^2$

mgf:
$$\psi_X(t) = \left(\frac{1}{1 - \beta t} \right)^\alpha \quad \text{for } t < \frac{1}{\beta}$$

(5) *The Chi Squared Distribution*

$X \sim \chi^2(p)$ for $p = 1, 2, 3, \dots$

pdf:
$$f_X(x) = \frac{1}{\Gamma(p/2)2^{p/2}} x^{(p/2)-1} e^{-x/2} \quad \text{for } 0 < x < \infty$$

and zero elsewhere

Expected value: $E(X) = p$

Variance: $\text{Var}(X) = 2p$

mgf:
$$\psi_X(t) = \left(\frac{1}{1 - 2t} \right)^{p/2} \quad \text{for } t < \frac{1}{2}$$