Math 29
Worksheet 6
Psychology

In a psychophysical experiment designed to measure performance in a recognition task, a subject is presented with a set of pictures of people's faces. Later, the subject is presented with a second set of pictures which contains the previously shown pictures and some new ones. The subject then is asked to answer "yes" or "no" to the question "Do you recognize this face?" We would like to determine a measure of the observer's ability to discriminate between the previously shown pictures and the new ones.

If a subject correctly recognizes a face as being one of the previously shown ones, it is called a "hit." If a subject incorrectly states that they recognize a face, when the face is actually a new one, it is called a "false alarm." The proportion of responses to previously shown faces which are hits is denoted by H, while the proportion of responses to new faces which are false alarms is denoted by F.

A measure of the ability of the subject to discriminate between previously shown faces and new ones is given by

$$s = \frac{1}{2}\left\{\log_{10}\left(\frac{H}{1-H}\right) - \log_{10}\left(\frac{F}{1-F}\right)\right\}.$$

1. Suppose that the subject responds "yes" to 20 and "no" to 5 of the previously shown pictures, while the subject gives 10 "yes" and 15 "no" responses to the new pictures. Compute H and F.

2. What do the numbers $1-H$ and $1-F$ represent?

3. Compute s for the pair (H, F) obtained in Problem 1 above as well as for the pairs $(H, F) = (0.80, 0.20)$, $(H, F) = (0.8, 0.6)$ and $(H, F) = (0.80, 0.90)$.

4. Another measure of sensitivity is given by $\alpha = 10^s$. Show that

$$\alpha = \sqrt{\frac{H(1-F)}{F(1-H)}}.$$

5. Suppose that $(H, F) = (0.6, 0.2)$. If F is unchanged, what would H have to be to double α? If H is unchanged, what would F have to be to double α?

6. An *unbiased* observer is one for which $H = 1 - F$. Give the simplest possible formulas for computing the measures of sensitivity s and α for an unbiased subject.

7. The proportion of correct answers given by a subject, i.e., "yes" to a previously shown face and "no" to a new one, is called the *percent correct* and is denoted by $p(c)$. Compute $p(c)$ for the experimental data given at the beginning of this worksheet. Given that $(H, F) = (0.8, 0.2)$ for an unbiased observer, compute $p(c)$.

8. Show that for an unbiased observer $p(c) = H$ and $\frac{\alpha}{1+\alpha} = H$, and conclude that $p(c) = \frac{\alpha}{1+\alpha}$.