Math 29
Worksheet 7
Population Growth

This worksheet will make use of the following exponential model for population growth:

$$N(t) = N(t_o)e^{r(t-t_o)}, \tag{1}$$

where $N(t)$ denotes the number of individuals in a given population at time t, the number r is called the rate of growth, and t_o is a fixed point in history.

1a) Use equation (1) to derive the following equation

$$\ln[N(t)] = \ln[N(t_o)] + r(t - t_o). \tag{2}$$

b) Below we give estimates of the world population from 1990 to 1999 in billions:

t =**year**	$N(t)$ =**global population in billions**
1990	5.25261
1991	5.33776
1992	5.41819
1993	5.50444
1994	5.58609
1995	5.66877
1996	5.75096
1997	5.83119
1998	5.91228
1999	5.99617

Plot the points $(t, \ln(N(t)))$ for the population data given above. Sketch the straight line that best fits the data points that you plotted. Assuming that the world population grew according to the exponential model with $t_o = 1990$, use your line to estimate the rate of growth r.

2. Using the exponential model for population growth with the rate you obtained in problem 1, how many times will the world population have doubled between 1990 and 2366?

3. Assume that the world's population in the decade of the 90's grew exponentially with a rate of growth, r, of 1.5% per year, and suppose that it continues to grow exponentially at that rate into the 21$^{\text{st}}$ century. Use the exponential model of population growth given by equation (1) to answer the following questions.

(a) What will the world's human population be in the year 2020?

(b) When will the total number of people living on Earth double the world's human population in 1999?

(c) Given that the total area of the continents is about $1.48 \times 10^{14}\text{m}^2$, how long will it take for the population density to reach 10^4people/km^2 (this is a typical density of a large city).

4. Suppose that the US population was 227 million in 1980 and 249 million in 1990. According to the exponential model for population growth, what will the US population be in the year 2020?

5. Suppose that a single cell of the bacterium *E. coli* divides every twenty minutes. Given that the average mass of an *E. coli* bacterium is 10^{-12} gr, if a cell of *E. coli* was allowed to reproduce without restrain to produce a super–colony, how long would it be until the total mass of the bacterial colony would be that of the earth (5.9763×10^{24} kg)?