Math 29
Worksheet 8
Art Forgery

Radioactive substances decay according to the formula

$$N(t) = N_o e^{-\lambda(t-t_o)},$$

where t_0 is the time you start measuring, $N(t)$ denotes the number of atoms of the element after t years, $N_o = N(t_o)$, and the number of atoms which disintegrate per year at time t is the quantity $\lambda N(t)$. We call λ the *decay constant*.

1a) Let T denote the half-life of the substance in years. Show that

$$\lambda = \frac{\ln 2}{T},$$

b) An ore of uranium–238 contains lead, and as the uranium–238 decays it causes a small amount of the lead to change into radioactive lead–210. The lead–210 and uranium–238 are in a state of radioactive equilibrium, where the number of disintegrations of lead-210 per minute per gram of Pb (ordinary lead) is the same as the number of disintegrations of uranium–238 per minute per gram of Pb. The half–life of uranium–238 is 4.51 billion years. Suppose that a Geiger counter detects 100 disintegrations of lead–210 per minute per gram of ordinary lead. How many atoms of uranium–238 per gram of Pb are present in the ore?

c) Using the fact that one mole of uranium–238 weighs 238 grams, and that there are 6.02×10^{23} atoms in a mole, compute the concentration of uranium in the ore of part b) above (in grams of uranium per gram of Pb). Express your answer as a percentage.

d) Suppose the uranium-238 content of an ore is 3% (per gram of Pb). What disintegration rate of lead–210 should be observed per minute per gram of Pb?

2. The half–life of uranium–238 is 4.51 billion years, while that of uranium–235 is 0.707 billion years. Suppose that a sample contains equal amounts of uranium–238 and uranium–235 at the time of its formation. If the proportion of uranium–238 to uranium–235 in the sample is currently 137.8 to 1, what is the age of the sample?

3. All paintings contain a small amount of the radioactive element lead–210, and an even smaller amount of radium–226, since these elements are contained in white lead, a pigment that artists have used for over 2000 years. Let $y(t)$ be the amount of lead–210 per gram of white lead present at time t, and let y_o be the amount of lead–210 per gram of white lead present at the time t_o of manufacture. Because the half-life of radium–226 is very large, for a period of time up to 300 years, the number of atoms of radium–226 which disintegrate per year per gram of white lead may be assumed to be a constant r. If λ is the decay constant for lead–210, then we have the equation

$$y(t) = \frac{r}{\lambda}(1 - e^{-\lambda(t-t_o)}) + y_o e^{-\lambda(t-t_o)}$$

a) Use the above equation to derive a formula for the age of a painting that is no more than three centuries old. Explain which terms of your formula can be figured out mathematically and/or experimentally, and which ones cannot?

b) Find a formula for computing λy_o (the number of atoms of lead–210 that disintegrate per year per gram of white lead at the time of manufacture of white lead) in terms of $\lambda y(t)$ (the number of atoms of lead–210 that disintegrate per year per gram of white lead currently), r (the number of atoms of radium that disintegrate per year per gram of white lead), λ, and the age of the painting.

c) The disintegration rate of lead–210 per gram of white lead at the time of manufacture of the white lead is the same as that found in ores that contain it, but decreases as the lead–210 in the white lead decays. The disintegration rate of lead–210 cannot exceed 25,000 disintegrations per minute per gram of white lead. This information can be used to determine if a painting purportedly made at the beginning of the 18^{th} century is a modern forgery or not. Suppose the painting is found to emit 8.5 disintegrations of lead–210 per minute per gram of white lead and 0.8 disintegrations of radium per minute per gram of white lead. Use the formula derived in part b) above to conclude that the painting cannot be 300 years old, and is therefore a forgery. The half–life of lead–210 is 22 years.